The current version of the DE and Mathematical Functions packages under development at Maplesoft, that is, dsolve, pdsolve, DEtools, PDEtools, DifferentialAlgebra and DifferentialGeometry, the mathematical functions of the language, the conversion network for mathematical functions and the MathematicalFunctions package, has this datestamp:

> restart; *DEsAndMathematicalFunctions:-Version()*[2];

2013, Dezember 10, 12:40 hours

If you installed *DEsAndMathematicalFunctions.mla* successfully (see installation instructions in the zip containing *DEsAndMathematicalFunctions.mla*) you will see the date shown above. The following computations are reproducible only with this current version.

December 10

• Implement the ability to hande a differential equation system in Matrix notation, including initial conditions and the testing of solutions using odetest. (See this <u>post in Mapleprimes from October</u> 29)

> restart;

> ode := Vector([diff(x(t), t), diff(y(t), t)]) = Vector([2 * x(t) + y(t), 3 * y(t) - x(t)]);

$$ode := \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 2x(t) + y(t) \\ 3y(t) - x(t) \end{bmatrix}$$
(1.1)

New: dsolve handles this system of equations represented as a "Vector" equation

$$\begin{cases} x(t) = e^{\frac{5t}{2}} \left(\sin\left(\frac{\sqrt{3}t}{2}\right) _CI + \cos\left(\frac{\sqrt{3}t}{2}\right) _C2 \right), y(t) = \\ -\frac{1}{2} \left(e^{\frac{5t}{2}} \left(\sin\left(\frac{\sqrt{3}t}{2}\right) \sqrt{3} _C2 - \cos\left(\frac{\sqrt{3}t}{2}\right) \sqrt{3} _CI - \sin\left(\frac{\sqrt{3}t}{2}\right) _CI \\ -\cos\left(\frac{\sqrt{3}t}{2}\right) _C2 \right) \right) \end{cases}$$

$$(1.2)$$

New: if the DE system is in vector notation, so is it the output of odetest > *odetest*((1.2), (1.1))

 $\begin{bmatrix} 0\\0 \end{bmatrix}$ (1.3)

New: the variables can also be specified in Vector notation, in which case dsolve's output will also be in Vector notation

>
$$V := Vector([x(t), y(t)])$$

 $V := \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$
(1.4)
> $sol := dsolve(ode, V)$

(1.5)

(1)

$$sol := \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \left[\left[\frac{1}{2} \left(e^{\frac{5t}{2}} \left(\sin\left(\frac{\sqrt{3}t}{2}\right) \sqrt{3} C^2 - \cos\left(\frac{\sqrt{3}t}{2}\right) \sqrt{3} C^2 \right] + \sin\left(\frac{\sqrt{3}t}{2}\right) C^2 + \cos\left(\frac{\sqrt{3}t}{2}\right) C^2 \right] \right],$$

$$\left[e^{\frac{5t}{2}} \left(\sin\left(\frac{\sqrt{3}t}{2}\right) C^2 + \cos\left(\frac{\sqrt{3}t}{2}\right) C^2 \right) \right],$$

$$\left[e^{\frac{5t}{2}} \left(\sin\left(\frac{\sqrt{3}t}{2}\right) C^2 + \cos\left(\frac{\sqrt{3}t}{2}\right) C^2 \right) \right] \right]$$
New: odetest can test this kind of Vector solution and returns in the same notation

> odetest(sol, ode)

$$\begin{bmatrix} 0\\0 \end{bmatrix}$$
(1.6)

The same is implemented for pdsolve and pdetest.

New: for ODEs, the initial conditions can also be given in Vector notation, using eval or D to represent the value of the functions or their derivatives at a given point. Example:

$$ICs := Vector([Eval(diff(x(t), t), t=0), Eval(diff(y(t), t), t=0)]) = Vector([1, 0]);$$

$$ICs := \begin{bmatrix} \dot{x}(t) \\ | t=0 \\ \dot{y}(t) | \\ | t=0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
(1.7)

[Here sys can be a set or a list with the ODEs and the initial conditions, say > sys := [ode, ICs]

$$sys := \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 2x(t) + y(t) \\ 3y(t) - x(t) \end{bmatrix}, \begin{bmatrix} \dot{x}(t) \\ t = 0 \\ \dot{y}(t) \\ t = 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
(1.8)

As in the previous case, if you pass the system without specifying the unknowns as a Vector, for instance without specifying them at all (in an example like this one there is no ambiguity and so they are not necessary), you have the system's solution returned as usual, as a set > dsolve([ode, ICs])

$$\begin{cases} x(t) = e^{\frac{5t}{2}} \left(-\frac{\sin\left(\frac{\sqrt{3}t}{2}\right)\sqrt{3}}{21} + \frac{3\cos\left(\frac{\sqrt{3}t}{2}\right)}{7} \right), y(t) = \\ -\frac{e^{\frac{5t}{2}} \left(\frac{10\sin\left(\frac{\sqrt{3}t}{2}\right)\sqrt{3}}{21} - \frac{2\cos\left(\frac{\sqrt{3}t}{2}\right)}{7} \right)}{2} \right] \end{cases}$$
(1.9)

The odetesting works as in the previous example: it returns in the same way as when there are no initial conditions, so as a list of Vectors

> odetest((**1.9**), [ode, ICs])

 $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (1.10)

If however you specify the unknowns as a Vector, then the output is also a Vector equation *dsolve*([*ode*, *ICs*], *V*)

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{5t}{2} \left(-\frac{2\sin\left(\frac{\sqrt{3}t}{2}\right)\sqrt{3}}{21} + \frac{6\cos\left(\frac{\sqrt{3}t}{2}\right)}{7} \right) \\ \frac{5t}{2} \left(-\frac{5\sin\left(\frac{\sqrt{3}t}{2}\right)\sqrt{3}}{21} + \frac{\cos\left(\frac{\sqrt{3}t}{2}\right)}{7} \right) \end{bmatrix}$$
(1.11)

• odetest((1.11), [ode, ICs])
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix}$$
(1.12)