$restart : with(LinearAlgebra) : with(plots) :$

4) Using transition matrices from one orthonormal basis to another.

a) Find parametric equations for the right circular cylinder having radius 3, length 12 whose axis is the z-axis and whose bottome edge lies in the plane: z = 0.

Soln:

$\therefore$ Parametric equations for the right circular cylinder:

$$x = 3\cos t \qquad y = 3\sin t \qquad 0 \le t \le 2\pi$$
$$z = s \qquad 0 \le s \le 12 \quad .$$

b) Find parametric equations (x, y, z) for the circular cylinder having radius 3, length 12, whose axis is the line through the origin that is perpendicular to the plane Π: x + 2y +2z = 0, and whose "bottom edge" lies in the plane Π, by following steps 1-4.

1) Find an orthonormal basis B" = $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ for $\Re^3$, where $\mathbf{w}_3$ lies along the axis of the cylinder.

Soln: Let V = {(x,y,z) r x + 2y +2z = 0}

V = NS(A), where A is the matrix

$$A := \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}:$$

$B := NullSpace(A);$

$$\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{1}$$

Now I will transform B into and orthonormal basis B" for V.

$ONB := GramSchmidt(B, normalized);$

$$\left\{ \begin{bmatrix} -\frac{2}{5}\sqrt{5} \\ 0 \\ \frac{1}{5}\sqrt{5} \end{bmatrix}, \begin{bmatrix} -\frac{2}{15}\sqrt{5} \\ \frac{1}{3}\sqrt{5} \\ -\frac{4}{15}\sqrt{5} \end{bmatrix} \right\} \tag{2}$$

Follow up: With **w1** and **w2**

$w1 := ONB[1];\ w2 := ONB[2];$

$$\begin{bmatrix} -\frac{2}{5}\sqrt{5} \\ 0 \\ \frac{1}{5}\sqrt{5} \end{bmatrix}$$

$$\begin{bmatrix} -\frac{2}{15}\sqrt{5} \\ \frac{1}{3}\sqrt{5} \\ -\frac{4}{15}\sqrt{5} \end{bmatrix} \qquad \textbf{(3)}$$

Let **w3 = w1 x w2** as to form an orthonormal basis B'' = $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$

$w3 := simplify(w1 \ \&x\ w2);$

$$\begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} \qquad \textbf{(4)}$$

r $B'' := (w1, w2, w3);$

$$\begin{bmatrix} -\frac{2}{5}\sqrt{5} \\ 0 \\ \frac{1}{5}\sqrt{5} \end{bmatrix}, \begin{bmatrix} -\frac{2}{15}\sqrt{5} \\ \frac{1}{3}\sqrt{5} \\ -\frac{4}{15}\sqrt{5} \end{bmatrix}, \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix} \qquad \textbf{(5)}$$

2) Find parametric equations (X, Y, Z) for the cylinder with respect to the basis B'' = $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$

Soln: Let C1 be a 3 x 1 column matrix whose rows contain parametric equation for the right circular cylinder whose axis is **w3**.

r $C1 := \begin{bmatrix} 3\cdot\cos(t) \\ 3\cdot\sin(t) \\ s \end{bmatrix};$

$$\begin{bmatrix} 3\cos(t) \\ 3\sin(t) \\ s \end{bmatrix} \tag{6}$$

where $0 \le t \le 2\pi$ & $0 \le s \le 12$

3) Find the transition matrix *P* from the basis B" to the standard basis *S*

Soln:

$P := \langle w1 \mid w2 \mid w3 \rangle;$

$$\begin{bmatrix} -\frac{2}{5}\sqrt{5} & -\frac{2}{15}\sqrt{5} & -\frac{1}{3} \\ 0 & \frac{1}{3}\sqrt{5} & -\frac{2}{3} \\ \frac{1}{5}\sqrt{5} & -\frac{4}{15}\sqrt{5} & -\frac{2}{3} \end{bmatrix} \tag{7}$$

4) Find parametric equations (x, y, z) for the cylinder with respect to the standard basis *S*.

Soln:

$C2 := P.C1;$

$$\begin{bmatrix} -\frac{6}{5}\sqrt{5}\cos(t) - \frac{2}{5}\sqrt{5}\sin(t) - \frac{1}{3}s \\ \sqrt{5}\sin(t) - \frac{2}{3}s \\ \frac{3}{5}\sqrt{5}\cos(t) - \frac{4}{5}\sqrt{5}\sin(t) - \frac{2}{3}s \end{bmatrix} \tag{8}$$

where $0 \le t \le 2\pi$ & $0 \le s \le 12$

c)

5) Graph the cylinder together with an axis line fo the cylinder and the plane Π.

Soln:

$Rightcylinder := spacecurve([C2[1], C2[2], C2[3]], t = 0..2 \cdot \mathrm{Pi}, s = 0..12, numpoints = 4000, thickness = 2, scaling = constrained, color = blue) :$

Warning, unable to evaluate the function to numeric values in the region; see the plotting command's help page to ensure the calling sequence is correct