http://www.mapleprimes.com/questions/200726-Numerical-Integration

```
> restart; interface(version);
  Digits:=15;
```

Classic Worksheet Interface, Maple 17.00, Windows, Feb 21 2013, Build ID 813473

Digits := 15

the task

```
> J1:=Int(Int(Int(max(0.,
  (0.9483573506e-3*(-1.*sin(a)*cos(w)-1.*cos(a)*sin(w)*sin(b)))*cos(a)*cos(b
  )^2*(200.*cos(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3+58.5*signum(-200
  .*cos(b)*sin(w)*sin(b)+1000.*k1+1000.*k2+1000.*k3))*Heaviside(-(1.*(200.*c
  os(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3))*signum(-200.*cos(b)*sin(w
  )*sin(b)+1000.*k1+1000.*k2+1000.*k3)-58.5)*Heaviside(1.-.98*cos(b)^2)/sqrt
  (1.-.98*cos(b)^2)), a = 0. .. 6.283185308), b = 0. .. 1.570796327), w = 0.
  .. 6.283185308)+Int(Int(Int(min(0.,
  (0.9483573506e-3*(-1.*sin(a)*cos(w)-1.*cos(a)*sin(w)*sin(b)))*cos(a)*cos(b
  )^2*(200.*cos(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3+58.5*signum(-200
  .*cos(b)*sin(w)*sin(b)+1000.*k1+1000.*k2+1000.*k3))*Heaviside(-(1.*(200.*c
  os(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3))*signum(-200.*cos(b)*sin(w
  )*sin(b)+1000.*k1+1000.*k2+1000.*k3)-58.5)*Heaviside(1.-.98*cos(b)^2)/sqrt
  (1.-.98*cos(b)^2)), a = 0. .. 6.283185308), b = 0. .. 1.570796327), w = 0.
  .. 6.283185308):

  J2:=Int(Int(Int(max(0.,
  (0.9483573506e-3*(cos(a)*cos(w)-1.*sin(a)*sin(b)*sin(w)))*sin(a)*cos(b)^2*
  (200.*cos(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3+58.5*signum(-200.*co
  s(b)*sin(w)*sin(b)+1000.*k1+1000.*k2+1000.*k3))*Heaviside(-(1.*(200.*cos(b
  )*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3))*signum(-200.*cos(b)*sin(w)*si
  n(b)+1000.*k1+1000.*k2+1000.*k3)-58.5)*Heaviside(1.-.98*cos(b)^2)/sqrt(1.-
  .98*cos(b)^2)), a = 0. .. 6.283185308), b = 0. .. 1.570796327), w = 0. ..
  6.283185308)+Int(Int(Int(min(0.,
  (0.9483573506e-3*(cos(a)*cos(w)-1.*sin(a)*sin(b)*sin(w)))*sin(a)*cos(b)^2*
  (200.*cos(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3+58.5*signum(-200.*co
  s(b)*sin(w)*sin(b)+1000.*k1+1000.*k2+1000.*k3))*Heaviside(-(1.*(200.*cos(b
  )*sin(w)*sin(b)-1000.*k1-1000.*k2-1000.*k3))*signum(-200.*cos(b)*sin(w)*si
  n(b)+1000.*k1+1000.*k2+1000.*k3)-58.5)*Heaviside(1.-.98*cos(b)^2)/sqrt(1.-
  .98*cos(b)^2)), a = 0. .. 6.283185308), b = 0. .. 1.570796327), w = 0. ..
  6.283185308):

  J3:=Int(Int(Int(max(0.,
  0.9483573506e-3*cos(b)^2*sin(w)*sin(b)*(200.*cos(b)*sin(w)*sin(b)-1000.*k1
  -1000.*k2-1000.*k3+58.5*signum(-200.*cos(b)*sin(w)*sin(b)+1000.*k1+1000.*k
  2+1000.*k3))*Heaviside(-(1.*(200.*cos(b)*sin(w)*sin(b)-1000.*k1-1000.*k2-1
  000.*k3))*signum(-200.*cos(b)*sin(w)*sin(b)+1000.*k1+1000.*k2+1000.*k3)-58
  .5)*Heaviside(1.-.98*cos(b)^2)/sqrt(1.-.98*cos(b)^2)), a = 0. ..
  6.283185308), b = 0. .. 1.570796327), w = 0. .. 6.283185308):
> map(identify, J1): convert(%, rational):
  combine(%, Int): combine(%, Int): combine(%, Int):
  convert(%, abs):
  JJ1:=simplify(%, size);

  map(identify, J2): convert(%, rational):
  combine(%, Int): combine(%, Int): combine(%, Int):
  convert(%, abs):
  JJ2:=simplify(%, size);

  map(identify, J3): convert(%, rational):
  combine(%, Int): combine(%, Int): combine(%, Int):
  convert(%, abs):
  JJ3:=simplify(%, size);
```

$$JJ1 := \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} \max\left(0, -\frac{21935280}{115648811}\cos(b)^2\right)$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)$$
$$(\sin(a)\cos(w)+\cos(a)\sin(w)\sin(b))\,(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\mathrm{Heaviside}\Big(-$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\Big)\cos(a)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)+\min\left(0, -\frac{21935280}{115648811}\cos(b)^2\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)$$
$$(\sin(a)\cos(w)+\cos(a)\sin(w)\sin(b))\,(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\mathrm{Heaviside}\Big(-$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\Big)\cos(a)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)\,da\,db\,dw$$

$$JJ2 := \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} \max\left(0, -\frac{21935280}{115648811}(\sin(a)\sin(b)\sin(w)-\cos(a)\cos(w))\cos(b)^2\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\sin(a)$$
$$(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\mathrm{Heaviside}\Big(-$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\Big)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)+\min\left(0, -\frac{21935280}{115648811}\right.$$

$$(\sin(a)\sin(b)\sin(w)-\cos(a)\cos(w))\cos(b)^2$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\sin(a)$$
$$(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)\,da\,db\,dw$$

$$JJ3:=\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\max\left(0,-\frac{109676400}{115648811}\sin(w)\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(b)^2$$
$$\left(-\frac{1}{5}\cos(b)\sin(w)\sin(b)+k1+k2+k3\right)\sin(b)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)+\min\left(0,-\frac{109676400}{115648811}\sin(w)\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(b)^2$$
$$\left(-\frac{1}{5}\cos(b)\sin(w)\sin(b)+k1+k2+k3\right)\sin(b)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)\,da\,db\,dw$$

```
> J_n = JJ_n;
```

$$J\_n = JJ\_n$$

```
>
> GetIntegrand(JJ1): GetIntegrand(%):
  A1:=GetIntegrand(%);

  GetIntegrand(JJ2): GetIntegrand(%):
  A2:=GetIntegrand(%);

  GetIntegrand(JJ3): GetIntegrand(%):
  A3:=GetIntegrand(%);
```

$$A1:=\max\left(0,-\frac{21935280}{115648811}\cos(b)^2\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\right.$$
$$(\sin(a)\cos(w)+\cos(a)\sin(w)\sin(b))\,(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(a)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)+\min\left(0,-\frac{21935280}{115648811}\cos(b)^2\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)$$
$$(\sin(a)\cos(w)+\cos(a)\sin(w)\sin(b))\,(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(a)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)$$

$$A2:=\max\left(0,-\frac{21935280}{115648811}(\sin(a)\sin(b)\sin(w)-\cos(a)\cos(w))\cos(b)^2\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\sin(a)$$
$$(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)+\min\left(0,-\frac{21935280}{115648811}\right.$$
$$(\sin(a)\sin(b)\sin(w)-\cos(a)\cos(w))\cos(b)^2$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\sin(a)$$
$$(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3)$$
$$(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3)\,/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\Big)$$

$$A3:=\max\left(0,-\frac{109676400}{115648811}\sin(w)\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\right.$$

$$\text{Heaviside}\left(-\left(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3\right)\right.$$
$$\left(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right)/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(b)^2$$
$$\left(-\frac{1}{5}\cos(b)\sin(w)\sin(b)+k1+k2+k3\right)\sin(b)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)+\min\left(0,-\frac{109676400}{115648811}\sin(w)\right.$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\text{Heaviside}\left(-\right.$$
$$\left(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3\right)$$
$$\left(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right)/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(b)^2$$
$$\left(-\frac{1}{5}\cos(b)\sin(w)\sin(b)+k1+k2+k3\right)\sin(b)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)\right)$$

```
> J_n = Int(Int(Int(A_n,a = 0 .. 2*Pi),b = 0 .. 1/2*Pi),w = 0 .. 2*Pi);
```

$$J\_n=\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} A\_n\,da\,db\,dw$$

eliminate max and min

```
> min(0,r)+max(0,r): %= convert(%, piecewise, r);
```

$$\min(0,r)+\max(0,r)=r$$

```
> tmp:='A3'; # example, works for all
  %:
  tmpMax:='select(has, tmp, max)';
  tmpMin:='select(has, tmp, min)';
  ``;
  'tmpMax=subs(min=max, tmpMin)'; %: is(%);
  'tmp=tmpMax+tmpMin'; is(%);
```

$$tmp := A3$$
$$tmpMax := \text{select}(\text{has}, tmp, \max)$$
$$tmpMin := \text{select}(\text{has}, tmp, \min)$$

$$tmpMax = \text{subs}(\min = \max, tmpMin)$$
$$\text{true}$$
$$tmp = tmpMax + tmpMin$$
$$\text{true}$$

```
> A1: select(has,%, max):
  B1:=op(2, %);
  ``;
  A2: select(has,%, max):
  B2:=op(2, %);
  ``;
  A3: select(has,%, max):
  B3:=op(2, %);
```

$$B1 := -\frac{21935280}{115648811}\cos(b)^2\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)$$
$$(\sin(a)\cos(w)+\cos(a)\sin(w)\sin(b))\,(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$\left(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3\right)$$
$$\left(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right)/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(a)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)$$

$$B2 := -\frac{21935280}{115648811}(\sin(a)\sin(b)\sin(w)-\cos(a)\cos(w))\cos(b)^2$$
$$\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\sin(a)$$
$$(\cos(b)\sin(w)\sin(b)-5\,k1-5\,k2-5\,k3)\,\text{Heaviside}\left(-\right.$$
$$\left(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3\right)$$
$$\left(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right)/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)$$

$$B3 := -\frac{109676400}{115648811}\sin(w)\left(\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\text{Heaviside}\left(\right.$$
$$-\left(200\cos(b)\sin(w)\sin(b)-1000\,k1-1000\,k2-1000\,k3\right)$$
$$\left(-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right)/$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|-\frac{117}{2}\right)\cos(b)^2$$
$$\left(-\frac{1}{5}\cos(b)\sin(w)\sin(b)+k1+k2+k3\right)\sin(b)\Big/\left(\sqrt{1-\frac{49}{50}\cos(b)^2}\right.$$
$$\left.\left|-200\cos(b)\sin(w)\sin(b)+1000\,k1+1000\,k2+1000\,k3\right|\right)$$

```
> J_n = Int(Int(Int(B_n,a = 0 .. 2*Pi),b = 0 .. 1/2*Pi),w = 0 .. 2*Pi);
```

$$J\_n=\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} B\_n\,da\,db\,dw$$

After some discussion and experiments guess (details omitted) the following, which eliminates parametes and reduces to prove
1. relations between the integrals
2. evaluating one of them

```
> 1000*k1+1000*k2+1000*k3; subs(k2=k1, k3=-2*k1, %);
```

$$1000\,k1+1000\,k2+1000\,k3$$
$$0$$

relations

```
> 'int(B1 - B2, a=0 ..2*Pi)';
    subs(k2=k1, k3=-2*k1, %): # activate that in case of doubts
  simplify(%) assuming 0<a,a<2*Pi, 0<b, b<Pi/2, 0<w,w<2*Pi: simplify(%,
  size);
```

$$\int_0^{2\pi} B1 - B2 \, da$$

$$0$$

```
> 'int(2*B2 + B3, a=0 .. 2*Pi)';
    subs(k2=k1, k3=-2*k1, %): # activate that in case of doubts
  simplify(%) assuming 0<a,a<2*Pi, 0<b, b<Pi/2, 0<w,w<2*Pi: simplify(%,
  size);
```

$$\int_0^{2\pi} 2\,B2 + B3 \, da$$

$$0$$

evaluating J3

```
> 'JJ3 = Int(Int(Int(A3,a = 0 .. 2*Pi),b = 0 .. 1/2*Pi),w = 0 .. 2*Pi)';
  is(%);
```

$$JJ3 = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} A3 \, da \, db \, dw$$

$$\text{true}$$

```
> 'Int(A3, [a=0 .. 2*Pi, b=0 .. Pi/2, w=0 .. 2*Pi], epsilon=1e-3,
  method=_cuhre)';
    subs(k2=k1, k3=-2*k1, %):
  evalf(%);
```

$$\text{Int}\left(A3, \left[a = 0 .. 2\pi, b = 0 .. \frac{\pi}{2}, w = 0 .. 2\pi\right], \varepsilon = 0.001, \text{method} = \_\text{cuhre}\right)$$

$$0.158489829771046$$

```
> # works better after kicking off max and min: one more decimal
  'Int(Int(Int(B3,a = 0 .. 2*Pi),b = 0 .. 1/2*Pi),w = 0 .. 2*Pi)';
    subs(k2=k1, k3=-2*k1, %):
  'Int(B3, [a=0 .. 2*Pi, b=0 .. Pi/2, w=0 .. 2*Pi], epsilon=1e-4,
  method=_cuhre)';
    subs(k2=k1, k3=-2*k1, %):
  evalf(%);
```

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{2\pi} B3 \, da \, db \, dw$$

$$\text{Int}\left(B3, \left[a = 0 .. 2\pi, b = 0 .. \frac{\pi}{2}, w = 0 .. 2\pi\right], \varepsilon = 0.0001, \text{method} = \_\text{cuhre}\right)$$

$$0.158396437448500$$

Now recognize, that for the 3rd integral there is 1) a trivial integration and 2) some symmetry. Giving an additional digit.

```
> 'indets(A3, symbol)': '%'=%;
  "variable  'a'  is missing ... fine!";
  ``;
```

```
  'int(const, a=0 .. 2*Pi)': '%'=%;
```

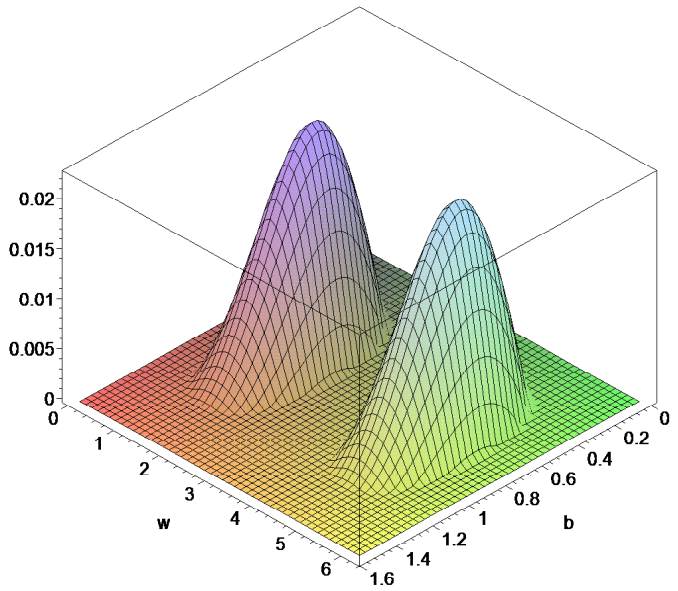
$$\text{indets}(A3, \text{symbol}) = \{b, k1, k2, k3, w\}$$

$$\text{"variable 'a' is missing ... fine!"}$$

$$\int_0^{2\pi} \text{const} \, da = 2 \, \text{const} \, \pi$$

```
> "symmetry for  'w' ";
  B3: subs(k2=k1, k3=-2*k1, %):
  plot3d(%, b=0 .. Pi/2, w=0 .. 2*Pi, axes=boxed);
  'B3 = eval(B3, w = w+Pi)'; subs(k2=k1, k3=-2*k1, %): simplify(%): is(%);
```

$$\text{"symmetry for 'w' "}$$



$$B3 = B3\Big|_{w = w + \pi}$$

$$\text{true}$$

```
> 2*2*Pi*'Int(B3, [b=0 .. Pi/2, w=0 .. Pi], epsilon=1e-5, method=_cuhre)';
    subs(k2=k1, k3=-2*k1, %):
  evalf(%);
```

$$4\pi\,\text{Int}\left(B3, \left[b = 0 .. \frac{\pi}{2}, w = 0 .. \pi\right], \varepsilon = 0.00001, \text{method} = \_\text{cuhre}\right)$$

$$0.158396548560801$$