

Case A

We have $a_1 \geq 2$, $a_2 \geq 2$ and $a_3 \geq 3$. and

$$f(a_1, a_2, a_3) = a_3(a_1-2) + a_1(a_3-2) + a_2(a_1-1) + a_1a_2 + \frac{a_1a_2(a_3-2) + 2}{a_3 - 1} + \frac{a_1a_3(a_3-1)}{1 + a_3(a_2 - 1)} + 2,$$

Cae B

We have $a_1 \geq 2$, $a_2 \geq 2$ and $a_3 \geq 2$

$$\begin{aligned} f(a_1, a_2, a_3) = & + \frac{a_1a_2(a_1-2)(a_3-2) + \frac{a_1a_3(a_2-2)}{2} + \frac{a_2(a_1a_3-4)}{2}}{1 - a_1 + a_1a_3} \\ & + \frac{a_2a_3(a_2-2)(a_1-2) + \frac{a_1a_2(a_3-2)}{2} + \frac{a_3(a_1a_2-4)}{2}}{1 - a_2 + a_1a_2} \\ & + \frac{a_1a_3(a_2-2)(a_3-2) + \frac{a_2a_3(a_1-2)}{2} + \frac{a_1(a_2a_3-4)}{2}}{1 - a_3 + a_2a_3} + 1, \end{aligned}$$