```
> # http://www.mapleprimes.com/questions/204401-Integral-With-Meijer-Functions
> b = 7/10;
```

$$b = \frac{7}{10}$$

```
> restart;
> Digits:=10; # for the original inputs
```

$$\text{Digits} := 10$$

```
> ms := (1.141448075+9.645873109*10^(-11)*I)*(MeijerG([[], []], [[1/4, 1/4],
  [1/2, 0]]], .3956862293*(-.70*x+1)^4)+(2.148399968-2.148399963*I)*MeijerG([[],
  []], [[1/4], [1/2, 1/4, 0]]],
  -.3956862293*(-.70*x+1)^4)+(-12.48809431-3.188863063*10^(-9)*I)*MeijerG([[],
  []], [[0], [1/2, 1/4, 1/4]]],
  -.3956862293*(-.70*x+1)^4)+(4.061500400*10^(-10)-8.649913391*I)*MeijerG([[],
  []], [[1/2], [1/4, 1/4, 0]]], -.3956862293*(-.70*x+1)^4));

  indets(ms, symbol);
```

$$\begin{aligned} ms := {} & (1.141448075 + 0.9645873109\,10^{-10}\,I)\left(\right. \\ & \text{MeijerG}\left([[\,],\,[\,]], \left[\left[\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, 0\right]\right], 0.3956862293\,(-0.70\,x+1)^4\right) \\ & + (2.148399968 - 2.148399963\,I)\,\text{MeijerG}\left([[\,],\,[\,]], \left[\left[\frac{1}{4}\right], \left[\frac{1}{2}, \frac{1}{4}, 0\right]\right], -0.3956862293\,(-0.70\,x+1)^4\right) \\ & - (12.48809431 + 0.3188863063\,10^{-8}\,I)\,\text{MeijerG}\left([[\,],\,[\,]], \left[[0], \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right]\right], -0.3956862293\,(-0.70\,x+1)^4\right) \\ & + (0.4061500400\,10^{-9} - 8.649913391\,I)\,\text{MeijerG}\left([[\,],\,[\,]], \left[\left[\frac{1}{2}\right], \left[\frac{1}{4}, \frac{1}{4}, 0\right]\right], -0.3956862293\,(-0.70\,x+1)^4\right) \\ & \left. \right) \end{aligned}$$

$$\{x\}$$

```
> ms: convert(%, rational):
  convert(%, StandardFunctions):
  #simplify(%) assuming 0<x,x<1: # ?
  h:=unapply(%, x);
```

$$\begin{aligned} h := x \to {} & \left(\frac{58441}{51199} + \frac{1}{10367127876}\,I\right)\left(\text{MeijerG}\left([[\,],\,[\,]], \left[\left[\frac{1}{4}, \frac{1}{4}\right], \left[\frac{1}{2}, 0\right]\right], \frac{2051}{51834000}\,(7\,x-10)^4\right) + \right. \\ & \left(\frac{52769}{3182866770} - \frac{23699}{1429452135}\,I\right)\sqrt{2}\;178521221958534315^{(1/4)}\,(-(7\,x-10)^4)^{(1/4)} \\ & \text{hypergeom}\left([\,], \left[\frac{3}{4}, 1, \frac{5}{4}\right], \frac{2051}{51834000}\,(7\,x-10)^4\right)/\pi \\ & - \frac{\left(\frac{159960}{12809} + \frac{1}{313591390}\,I\right)\text{hypergeom}\left([\,], \left[\frac{1}{2}, \frac{3}{4}, \frac{3}{4}\right], \frac{2051}{51834000}\,(7\,x-10)^4\right)}{\sqrt{\pi}\;\Gamma\left(\frac{3}{4}\right)^2} + \\ & \left.\left(\frac{4}{1595284835858625} - \frac{432784}{8104461875}\,I\right)\sqrt{265778835}\,\sqrt{-(7\,x-10)^4}\right. \end{aligned}$$

$$\text{hypergeom}\left([\,], \left[\frac{5}{4}, \frac{5}{4}, \frac{3}{2}\right], \frac{2051}{51834000}\,(7\,x-10)^4\right)\Gamma\left(\frac{3}{4}\right)^2 \Big/ \pi^{(5/2)}\Bigg)$$

```
> Digits:=15;
```

$$\text{Digits} := 15$$

```
> #H3p_Clean:= proc(ms::algebraic, b::numeric, eps::positive:=
  0.5*10^(1-Digits))
  H3p:= proc(ms::algebraic, b, eps::positive:= 0.5*10^(1-Digits))
  local x:= indets(ms, And(name, Not(constant))), D, X, Y;
       if nops(x)>1 then
            error "1st argument must be an expression of at most 1 variable"
       end if;
       x:= x[];
       D:= diff(ms,x);
  #     evalf(
       (
            subsindets(
                 Int(
                      diff(
                           subs(x= Y, D)*Int((-b*X+1)*Int(D^2, x= 0..X), X=
  1..Y),
                           Y
                      )*subs(x= Y, ms),
                      Y= 0..1
                 ),
                 specfunc(anything, Int),
  #               J-> Int(op(J), epsilon= eps)
                 J-> Int(op(J))
            )
       )
  end proc:
```

```
> H:=H3p(f(x), b, eps):
  H:=convert(H, D);
  #Expand(%);
```

$$H := \int_0^1 \left(\left(D^{(2)} \right)(f)(Y) \int_1^Y (-X\,b+1) \int_0^X D(f)(x)^2\,dx\,dX + D(f)(Y)\,(-Y\,b+1) \int_0^Y D(f)(x)^2\,dx \right) f(Y)\,dY$$

```
>
```

re-write over unit cube

```
> Int(diff(f(x),x)^2,x = 0 .. Y): convert(%, D);
  %=Change(%, x=Y*xi, xi):
  subs1:=subs(xi=x, %);
  #subs(subs1 , H); #applyrule(subs1 , H); # does not work?
```

$$\int_0^Y D(f)(x)^2\,dx$$

$$subs1 := \int_0^Y D(f)(x)^2\,dx = Y \int_0^1 D(f)(Y\,x)^2\,dx$$

```
> Int((`@@`(D,2)(f)(Y)*Int((-X*b+1)*Int(D(f)(x)^2,x = 0 .. X),X = 1 ..
  Y)+D(f)(Y)*(-Y*b+1)*Int(D(f)(x)^2,x = 0 .. Y))*f(Y),Y = 0 .. 1);
  J1:=Int((`@@`(D,2)(f)(Y)*Int((-X*b+1)*Int(D(f)(x)^2,x = 0 .. X),X = 1 ..
  Y)+D(f)(Y)*(-Y*b+1)*(Y*Int(D(f)(Y*x)^2,x = 0 .. 1)))*f(Y),Y = 0 .. 1);
  #combine(%, Int);
```

$$\int_0^1 \left((D^{(2)})(f)(Y) \int_1^Y (-X\,b+1) \int_0^X D(f)(x)^2\,dx\,dX + D(f)(Y)\,(-Y\,b+1) \int_0^Y D(f)(x)^2\,dx \right) f(Y)\,dY$$

$$J1 := \int_0^1 \left((D^{(2)})(f)(Y) \int_1^Y (-X\,b+1) \int_0^X D(f)(x)^2\,dx\,dX + D(f)(Y)\,(-Y\,b+1)\,Y \int_0^1 D(f)(Y\,x)^2\,dx \right) f(Y)\,dY$$

```
> Int(diff(f(x),x)^2,x = 0 .. X):
  %=Change(%, x=X*xi, xi):
  subs(xi=x, %);
```

$$\int_0^X \left(\frac{d}{dx} f(x) \right)^2 dx = \int_0^1 D(f)(X\,x)^2\,X\,dx$$

```
> Int((`@@`(D,2)(f)(Y)*Int((-X*b+1)*Int(D(f)(x)^2,x = 0 .. X),X = 1 ..
  Y)+D(f)(Y)*(-Y*b+1)*Y*Int(D(f)(Y*x)^2,x = 0 .. 1))*f(Y),Y = 0 .. 1);
  J2:=Int((`@@`(D,2)(f)(Y)*Int((-X*b+1)* (Int(D(f)(X*x)^2*X,x = 0 .. 1)),X =
  1 .. Y)+D(f)(Y)*(-Y*b+1)*Y*Int(D(f)(Y*x)^2,x = 0 .. 1))*f(Y),Y = 0 .. 1);
  #combine(%, Int);
```

$$\int_0^1 \left((D^{(2)})(f)(Y) \int_1^Y (-X\,b+1) \int_0^X D(f)(x)^2\,dx\,dX + D(f)(Y)\,(-Y\,b+1)\,Y \int_0^1 D(f)(Y\,x)^2\,dx \right) f(Y)\,d$$

$$Y$$

$$J2 := \int_0^1 \left((D^{(2)})(f)(Y) \int_1^Y (-X\,b+1) \int_0^1 D(f)(X\,x)^2\,X\,dx\,dX + D(f)(Y)\,(-Y\,b+1)\,Y \int_0^1 D(f)(Y\,x)^2\,dx \right) f(Y)$$

$$dY$$

```
> Int((-X*b+1)*Int(D(f)(X*x)^2*X,x = 0 .. 1),X = 1 .. Y);
  Change(%, {x=xi, X = (-1+Y)*Xi+1}, [xi, Xi]):
  subs(xi=x, Xi=X, %);
  #simplify(%);
```

$$\int_1^Y (-X\,b+1) \int_0^1 D(f)(X\,x)^2\,X\,dx\,dX$$

$$\int_0^1 -(X\,Y\,b - X\,b + b - 1) \int_0^1 D(f)((X\,Y - X + 1)\,x)^2\,(X\,Y - X + 1)\,dx\,(-1+Y)\,dX$$

```
> Int((`@@`(D,2)(f)(Y)*Int((-X*b+1)*Int(D(f)(X*x)^2*X,x = 0 .. 1),X = 1 ..
  Y)+D(f)(Y)*(-Y*b+1)*Y*Int(D(f)(Y*x)^2,x = 0 .. 1))*f(Y),Y = 0 .. 1);
  J3:=Int((`@@`(D,2)(f)(Y)*(Int(-(X*Y*b-X*b+b-1)*Int(D(f)((X*Y-X+1)*x)^2*(X*
  Y-X+1),x = 0 .. 1)*(Y-1),X = 0 .. 1))+D(f)(Y)*(-Y*b+1)*Y*Int(D(f)(Y*x)^2,x
  = 0 .. 1))*f(Y),Y = 0 .. 1);
  #simplify(%);
```

$$\int_0^1 \left((D^{(2)})(f)(Y) \int_1^Y (-X\,b+1) \int_0^1 D(f)(X\,x)^2\,X\,dx\,dX + D(f)(Y)\,(-Y\,b+1)\,Y \int_0^1 D(f)(Y\,x)^2\,dx \right) f(Y)$$

$$dY$$

$$J3 := \int_0^1 \left((D^{(2)})(f)(Y) \int_0^1 -(X\,Y\,b - X\,b + b - 1) \int_0^1 D(f)((X\,Y - X + 1)\,x)^2\,(X\,Y - X + 1)\,dx\,(-1 + Y)\,dX \right.$$

$$\left. + D(f)(Y)\,(-Y\,b + 1)\,Y \int_0^1 D(f)(Y\,x)^2\,dx \right) f(Y)\,dY$$

```
> 
> #X*Y-X+1; plot3d(%, X=0..1,Y=0..1, axes=boxed);
  0<= X*Y-X+1 and X*Y-X+1 <= 1; is(%) assuming 0<=X, 0<=Y, X<=1, Y<=1;
```

$$0 \le X\,Y - X + 1 \ \textbf{and}\ X\,Y - X \le 0$$

true

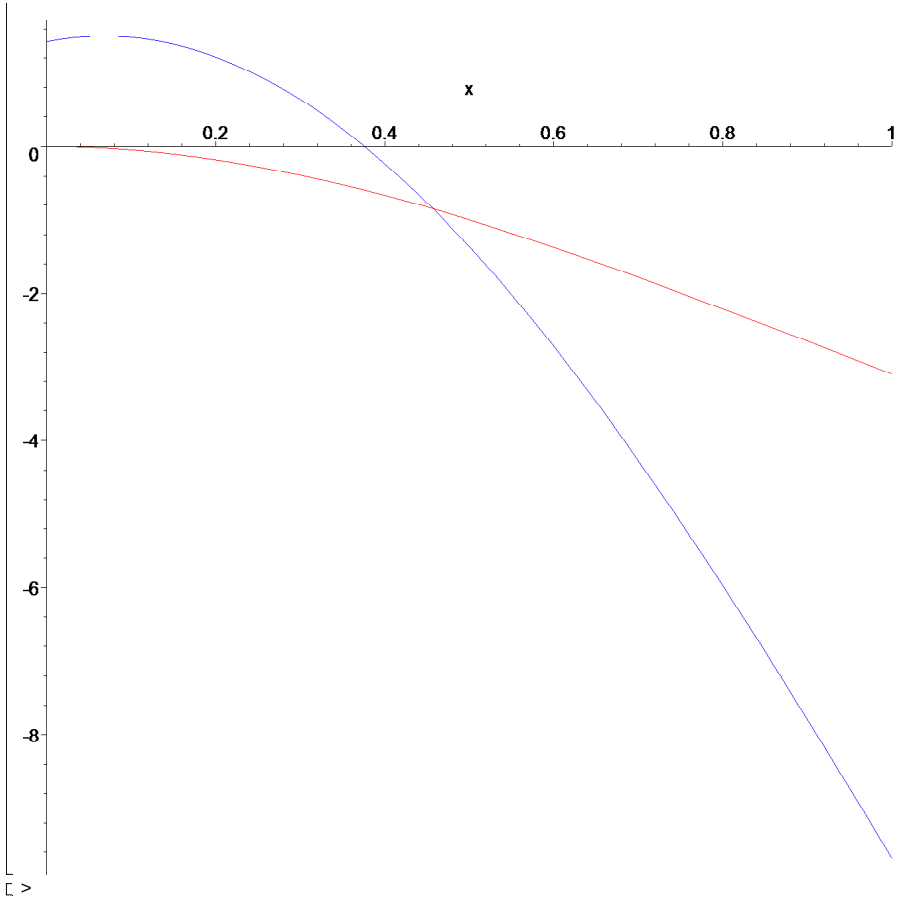
```
> 'eval(h(x), b=0.7)'; [eval(%, x=0), eval(%, x=1)]: evalf(%); #%*10^9;
```

$$h(x)\Big|_{b=0.7}$$

$$[-0.641458547493236\,10^{-7} + 0.141780658678275\,10^{-9}\,I,\ -3.09539451232627 - 0.968316944610845\,10^{-9}\,I]$$

```
> 'eval(h(x), b=0.7)';
  #'diff(%,x$2)';
  plot([Re(%), 10^10*Im(%)], x=0..1, color=[red,blue]);
```

$$h(x)\Big|_{b=0.7}$$




```
> 
```

Chebyshev

```
> with(numapprox);
```

[chebdeg, chebmult, chebpade, chebsort, chebyshev, confracform, hermite_pade, hornerform, infnorm, laurent, minimax, pade, remez]

```
> chebpade(exp(x), x=0..1, 5):
  eval(%, T = orthopoly[T]);
```

$$0.824361480821884 + 1.64872183351086\,x + 0.206069892612673\,(2\,x - 1)^2 + 0.0343461102350008\,(2\,x - 1)^3 + 0.00434749464920125\,(2\,x - 1)^4 + 0.000433846958609114\,(2\,x - 1)^5$$

```
> #chebyshev(exp(I*x), x=0..1, .001);
> 
> Digits:=32:
  rng:= 0 .. 1;
  eps:=1e-8;
  st:=time():
```

```
hh:=h;
chebyshev(x -> Re(hh(x)), x=rng, eps) + chebyshev(x-> Im(hh(x)), x= rng,
10^(-10)*eps)*I:
eval(%, T = orthopoly[T]): convert(%, rational, 15):
approx:=unapply(%, x);
``;
hh:=D(h):
chebyshev(x -> Re(hh(x)), x=rng, eps) + chebyshev(x-> Im(hh(x)), x= rng,
10^(-10)*eps)*I:
eval(%, T = orthopoly[T]): convert(%, rational, 15):
approx1:=unapply(%, x);
``;
hh:=(D@@2)(h):
chebyshev(x -> Re(hh(x)), x=rng, eps) + chebyshev(x-> Im(hh(x)), x= rng,
10^(-10)*eps)*I:
eval(%, T = orthopoly[T]): convert(%, rational, 15):
approx2:=unapply(%, x);

Digits:=15:
`seconds needed` = time() - st;
```

$$\mathrm{rng} := 0 .. 1$$

$$\mathrm{eps} := 0.1\ 10^{-7}$$

$$\mathrm{hh} := h$$

$$\begin{aligned}\mathrm{approx} := x \to & -\frac{562173}{161987786}(2x-1)^6 - \frac{106671}{309770140}(2x-1)^7 - \frac{191411}{700593199}(2x-1)^8 \\ & - \frac{118349}{937889945}(2x-1)^9 - \frac{80655}{2769083261}(2x-1)^{10} - \frac{25292}{2575567349}(2x-1)^{11} - \frac{44933618}{12607573}x + \frac{5886799}{7494263} \\ & - \frac{51883}{2378125609}(2x-1)^{12} - \frac{18471}{1714325266}(2x-1)^{13} + \left(- \frac{11}{11021108326941}(2x-1)^6 \right. \\ & + \frac{1}{33700868403345}(2x-1)^7 - \frac{1}{65895046183921}(2x-1)^8 - \frac{1}{87915560478044}(2x-1)^9 \\ & - \frac{1}{379855644022989}(2x-1)^{10} - \frac{1}{1217785151666296}(2x-1)^{11} - \frac{201}{160678172548}x + \frac{128}{261643766823} \\ & - \frac{1}{542648997535716}(2x-1)^{12} - \frac{1}{1098233214814659}(2x-1)^{13} - \frac{413}{1437887025858}(2x-1)^2 \\ & \left. + \frac{217}{2941359358090}(2x-1)^3 + \frac{34}{3026957493377}(2x-1)^4 - \frac{12}{3566390080673}(2x-1)^5 \right) I \\ & - \frac{8372599}{15184221}(2x-1)^2 + \frac{2956853}{11861225}(2x-1)^3 + \frac{1188173}{297023761}(2x-1)^4 - \frac{6114251}{422127421}(2x-1)^5\end{aligned}$$

$$\begin{aligned}\mathrm{approx1} := x \to & -\frac{9319415}{2112666}x - \frac{178756}{584064927}(2x-1)^{11} + \frac{14375}{1952899652}(2x-1)^{12} \\ & + \frac{137918}{7666903173}(2x-1)^{13} - \frac{144413}{1632512915}(2x-1)^{14} - \frac{89343}{1850937839}(2x-1)^{15} - \frac{4735080}{32693413}(2x-1)^4 \\ & - \frac{2305595}{55279439}(2x-1)^5 - \frac{255811}{52275230}(2x-1)^6 - \frac{424257}{102216097}(2x-1)^7 - \frac{300227}{146933083}(2x-1)^8 \\ & - \frac{459673}{491357977}(2x-1)^9 - \frac{3684211}{6341786550}(2x-1)^{10} + \frac{25708811}{17188213}(2x-1)^2 + \frac{2105634}{65781811}(2x-1)^3 + \left(\right. \\ & \left. - \frac{275}{119678851481}x - \frac{2}{77391146952921}(2x-1)^{11} + \frac{1}{1610943251786960}(2x-1)^{12} \right. \\ & + \frac{1}{65794272186901}(2x-1)^{13} - \frac{1}{133772095843704}(2x-1)^{14} - \frac{1}{245157963496383}(2x-1)^{15} \\ & - \frac{55}{1634640141154}(2x-1)^4 - \frac{51}{4256280965465}(2x-1)^5 + \frac{14}{34205802757281}(2x-1)^6 \\ & - \frac{3}{13383908988175}(2x-1)^7 - \frac{4}{21566929905679}(2x-1)^8 - \frac{2}{24248031725403}(2x-1)^9 \\ & - \frac{2}{40898239429403}(2x-1)^{10} + \frac{161}{363716521754}(2x-1)^2 + \frac{120}{1335413299117}(2x-1)^3 - \frac{3}{29400328859} \\ & \left. \right) I - \frac{8195987}{6033503}\end{aligned}$$

$$\begin{aligned}\mathrm{approx2} := x \to & \frac{54996913}{4596180}x - \frac{183185}{42252468}(2x-1)^{11} - \frac{647369}{245789462}(2x-1)^{12} + \frac{258221}{338717634}(2x-1)^{13} \\ & + \frac{607981}{1021548290}(2x-1)^{14} - \frac{155471}{167997417}(2x-1)^{15} - \frac{5968650}{14309599}(2x-1)^4 - \frac{4624775}{79063214}(2x-1)^5 \\ & - \frac{14027848}{242438893}(2x-1)^6 - \frac{1953034}{57635439}(2x-1)^7 - \frac{1037186}{57956559}(2x-1)^8 - \frac{509645}{60406499}(2x-1)^9 \\ & - \frac{331442}{77188771}(2x-1)^{10} + \frac{1387317}{7223450}(2x-1)^2 - \frac{5029396}{4340619}(2x-1)^3 - \frac{79111577}{7611197} \\ & - \frac{495854}{910745003}(2x-1)^{16} + \left(\frac{429}{121144708912}x - \frac{10}{55012751202039}(2x-1)^{11} \right. \\ & - \frac{4}{18145485818867}(2x-1)^{12} - \frac{5}{37230964956353}(2x-1)^{13} + \frac{2}{40842175474917}(2x-1)^{14} \\ & + \frac{1}{28332953232263}(2x-1)^{15} - \frac{190}{1585642226197}(2x-1)^4 + \frac{77}{15631244907508}(2x-1)^5 \\ & - \frac{11}{3528529981155}(2x-1)^6 - \frac{15}{4935804599098}(2x-1)^7 - \frac{19}{12077963140943}(2x-1)^8 \\ & - \frac{12}{14872676290919}(2x-1)^9 - \frac{9}{24711399676979}(2x-1)^{10} + \frac{219}{406188135938}(2x-1)^2 \\ & - \frac{191}{709579074879}(2x-1)^3 - \frac{122}{29987027685} - \frac{1}{21893377133762}(2x-1)^{16} \\ & \left. - \frac{1}{37461268922247}(2x-1)^{17} \right) I\end{aligned}$$

$$\mathrm{seconds\ needed} = 1.373$$

```
> degree( approx(x), x);  #degree( evalc(Re(approx(x))), x), degree(
evalc(Im(approx(x))), x);
degree( approx1(x), x); #degree( evalc(Re(approx1(x))), x), degree(
evalc(Im(approx1(x))), x);
degree( approx2(x), x); #degree( evalc(Re(approx2(x))), x), degree(
evalc(Im(approx2(x))), x);
```

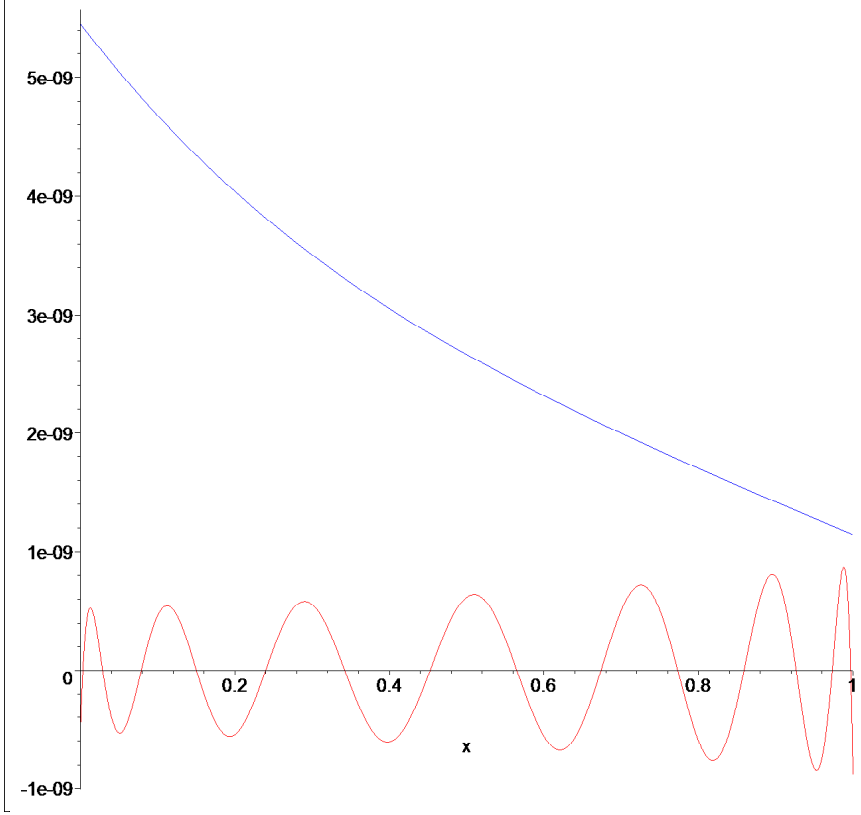
$$13$$

$$15$$

$$17$$

```
> 'h(x) - approx(x)';
#'diff(%,x$1)'; %:
plot([Re(%), 10^6*Im(%)], x=0..1, color=[red,blue]);
```

$$h(x) - \mathrm{approx}(x)$$




```
> 'D(h)(x) - approx1(x)';
  #plot([Re(%), 10^6*Im(%)], x=0..1, color=[red,blue]);
```

$$D(h)(x) - \text{approx1}(x)$$

```
> '(D@@2)(h)(x) - approx2(x)';
  #plot([Re(%), 10^6*Im(%)], x=0..1, color=[red,blue]);
```

$$(D^{(2)})(h)(x) - \text{approx2}(x)$$

```
>
>
> 'J3'; #combine(%);
  eval(%, (D@@2)(f)=`f ''`):
  eval(%, D(f)=`f '`);
  #indets(%); indets(%, symbol);
```

$$J3$$

$$\int_0^1 \left(f''(Y) \int_0^1 -(X\,Y\,b - X\,b + b - 1) \int_0^1 f'((X\,Y - X + 1)\,x)^2\,(X\,Y - X + 1)\,dx\,(-1 + Y)\,dX \right.$$

$$\left. + f'(Y)\,(-Y\,b + 1)\,Y \int_0^1 f'(Y\,x)^2\,dx \right] f(Y)\,dY$$

```
>
```

helper

to reduce complexity (length) of solution

```
> I1:=Int(D(f)(Y*x)^2,x = 0 .. 1); #eval(%, D(f)=w);
  eval(%, D(f)=approx1):
  value(%): simplify(%): evalf[2*Digits](%): convert(%, rational):
  collect(%, Y): #factor(%):
  r1:=I1=%;
```

$$I1 := \int_0^1 D(f)(Y\,x)^2\,dx$$

$$r1 := \int_0^1 D(f)(Y\,x)^2\,dx = \frac{1}{42423071090041788} - \frac{1}{448053323914448567}\,I$$
$$+ \left(\frac{3212071}{39802428} + \frac{13}{953130765645}\,I\right) Y^{30} - \left(\frac{10399747}{9471278} + \frac{95}{511910559212}\,I\right) Y^{29}$$
$$+ \left(\frac{39907265}{5585037} + \frac{203}{168094954018}\,I\right) Y^{28} - \left(\frac{63856284}{2160257} + \frac{199}{39832651707}\,I\right) Y^{27}$$
$$+ \left(\frac{73665115}{844823} + \frac{1173}{79595069966}\,I\right) Y^{26} - \left(\frac{188014816}{963903} + \frac{2439}{73983954079}\,I\right) Y^{25}$$
$$+ \left(\frac{652869694}{1899921} + \frac{2015}{34695007547}\,I\right) Y^{24} - \left(\frac{1051803526}{2151743} + \frac{38647}{467781775497}\,I\right) Y^{23}$$
$$+ \left(\frac{213690431}{374093} + \frac{11695}{121126194733}\,I\right) Y^{22} - \left(\frac{242873845}{437558} + \frac{17775}{189443055803}\,I\right) Y^{21}$$
$$+ \left(\frac{128896811}{284799} + \frac{2549}{33299927514}\,I\right) Y^{20} - \left(\frac{571251077}{1826624} + \frac{1848}{34709053639}\,I\right) Y^{19}$$
$$+ \left(\frac{132160851}{707851} + \frac{2249}{68202218225}\,I\right) Y^{18} - \left(\frac{52830392}{525883} + \frac{1919}{96659086154}\,I\right) Y^{17}$$
$$+ \left(\frac{70897213}{1290402} + \frac{1511}{114698586104}\,I\right) Y^{16} - \left(\frac{69082253}{1513297} + \frac{1101}{83103617686}\,I\right) Y^{15}$$
$$+ \left(\frac{38024047}{553566} + \frac{1047}{46121986262}\,I\right) Y^{14} - \left(\frac{35747922}{339013} + \frac{3142}{82002589427}\,I\right) Y^{13}$$
$$+ \left(\frac{62870273}{509985} + \frac{903}{18714939997}\,I\right) Y^{12} - \left(\frac{19435142}{184677} + \frac{4181}{95427343870}\,I\right) Y^{11}$$

$$+\left(\frac{55762988}{845331}+\frac{6023}{206955794127}\,I\right)Y^{10}-\left(\frac{55932749}{1842539}+\frac{1067}{74833329389}\,I\right)Y^{9}$$
$$+\left(\frac{20588541}{2120284}+\frac{499}{106720937733}\,I\right)Y^{8}-\left(\frac{25118363}{7025447}+\frac{479}{310038710213}\,I\right)Y^{7}$$
$$+\left(\frac{15436129}{2765034}+\frac{652}{168483134465}\,I\right)Y^{6}-\left(\frac{19591120}{25486531}+\frac{269}{161258583785}\,I\right)Y^{5}$$
$$-\left(\frac{12788680}{3386023}+\frac{2569}{566764256932}\,I\right)Y^{4}-\left(\frac{24944773}{1429004}+\frac{821}{103604131450}\,I\right)Y^{3}$$
$$+\left(\frac{68201447}{2297069}+\frac{1289}{58938616256}\,I\right)Y^{2}+\left(\frac{1138}{24807777943}-\frac{232}{107068392063}\,I\right)Y$$

```
> 
> 
> I2:=Int(D(f)((X*Y-X+1)*x)^2*(X*Y-X+1),x = 0 .. 1);
  simplify(%);
  eval(%, D(f)=approx1):
  value(%): simplify(%): simplify(%, size): evalf[2*Digits](%): convert(%,
  rational): # collect(%, [X,Y]):
  r2:=I2=%;
```

$$I2 := \int_0^1 D(f)((X\,Y-X+1)\,x)^2\,(X\,Y-X+1)\,dx$$

$$(X\,Y-X+1)\int_0^1 D(f)((X\,Y-X+1)\,x)^2\,dx$$

$$r2 := \int_0^1 D(f)((X\,Y-X+1)\,x)^2\,(X\,Y-X+1)\,dx = 13\,(1+(-1+Y)\,X)\left(\frac{2500810305551}{3}+\frac{254786024}{404505}\,I\right.$$
$$+\left(-\frac{5514295345186}{9}-\frac{230646853}{374361}\,I+\left(\frac{5514295345186}{9}+\frac{230646853}{374361}\,I\right)Y\right)X$$
$$+\left(\frac{55556370987473}{4}+\frac{598263926}{254861}\,I\right)(-1+Y)^{26}\,X^{26}$$
$$+\left(\frac{80609110134986}{21}+\frac{173312762}{267147}\,I\right)(-1+Y)^{27}\,X^{27}$$
$$+\left(38557886045102+\frac{395440780}{60681}\,I\right)(-1+Y)^{25}\,X^{25}$$
$$+\left(85584932515329+\frac{984546954}{68065}\,I\right)(-1+Y)^{24}\,X^{24}+\left(\frac{3976918841571}{41}+\frac{14732795}{898684}\,I\right)(-1+Y)^{29}\,X^{29}$$
$$+\left(\frac{6104251387833}{8}+\frac{99560833}{772024}\,I\right)(-1+Y)^{28}\,X^{28}+\left(\frac{520675783233}{88}+I\right)(-1+Y)^{30}\,X^{30}$$
$$+\left(\frac{10009434786034}{7}+\frac{271500526}{279277}\,I\right)(-1+Y)^{4}\,X^{4}+\left(\frac{3063439809419}{5}+\frac{158959617}{977921}\,I\right)(-1+Y)^{3}\,X^{3}$$
$$-\left(\frac{16542891272440}{27}+\frac{231501101}{470662}\,I\right)(-1+Y)^{2}\,X^{2}+\left(\frac{24138121453162}{3}+\frac{380021997}{125125}\,I\right)(-1+Y)^{8}\,X^{8}$$
$$+\left(\frac{14123082438962}{3}+\frac{334137361}{175114}\,I\right)(-1+Y)^{7}\,X^{7}+\left(\frac{25474565760821}{11}+\frac{318727825}{351163}\,I\right)(-1+Y)^{6}\,X^{6}$$
$$+\left(\frac{9523959992669}{9}+\frac{239625828}{435127}\,I\right)(-1+Y)^{5}\,X^{5}+\left(77235168794195+\frac{755452334}{55373}\,I\right)(-1+Y)^{13}\,X^{13}$$
$$+\left(49814996678897+\frac{1073265747}{113761}\,I\right)(-1+Y)^{12}\,X^{12}$$
$$+\left(\frac{63711408946527}{2}+\frac{295779950}{41429}\,I\right)(-1+Y)^{11}\,X^{11}$$
$$+\left(\frac{41053088566613}{2}+\frac{797327341}{140038}\,I\right)(-1+Y)^{10}\,X^{10}+\left(\frac{39217781377196}{3}+\frac{404926767}{93407}\,I\right)(-1+Y)^{9}\,X^{9}$$
$$+\left(283942715521199+\frac{2204936890}{45687}\,I\right)(-1+Y)^{17}\,X^{17}$$
$$+\left(221795995734423+\frac{1308004035}{34357}\,I\right)(-1+Y)^{16}\,X^{16}$$
$$+\left(164015743628969+\frac{2195090359}{77101}\,I\right)(-1+Y)^{15}\,X^{15}$$
$$+\left(115379831258394+\frac{1469296577}{72958}\,I\right)(-1+Y)^{14}\,X^{14}$$
$$+\left(339095332016633+\frac{13934964328}{242865}\,I\right)(-1+Y)^{18}\,X^{18}$$
$$+\left(364490553536436+\frac{5590931187}{90754}\,I\right)(-1+Y)^{20}\,X^{20}$$
$$+\left(370719517695846+\frac{2185027747}{34870}\,I\right)(-1+Y)^{19}\,X^{19}$$
$$+\left(240519417870147+\frac{5621284621}{138281}\,I\right)(-1+Y)^{22}\,X^{22}$$
$$+\left(317172710428863+\frac{3523598111}{65729}\,I\right)(-1+Y)^{21}\,X^{21}$$
$$+\left(156379511124382+\frac{6637273042}{251127}\,I\right)(-1+Y)^{23}\,X^{23}\Big)\Big/\,953130765645$$

```
> 
> 
> subs(r1, J3):
  subs(r2, %):

  eval(%, (D@@2)(f)=approx2):
  eval(%, D(f)=approx1):
  eval(%, f=approx): #length(%);

  value(%): simplify(%): collect(%, b):
  evalf(%);
  Sol:=unapply(%, b);
```

$$26.0856584874641 + 0.375109181142440\ 10^{-7}\,I - (21.9402270132279 + 0.317896416449617\ 10^{-7}\,I)\,b$$

$$Sol := b \to 26.0856584874641 + 0.375109181142440\ 10^{-7}\,I - (21.9402270132279 + 0.317896416449617\ 10^{-7}\,I)\,b$$

```
> 'Sol(7/10)': '%'= evalf(%);
  'Sol(-0.9)': '%'= evalf(%);
```

$$Sol\left(\frac{7}{10}\right) = 10.7274995782046 + 0.152581689627708\ 10^{-7}\,I$$

$$Sol(-0.9) = 45.8318627993692 + 0.661215955947095\ 10^{-7}\,I$$

```
> 
```