```
> # http://www.kurims.kyoto-u.ac.jp/~ooura/papers/DE-FT-gen.pdf, complex
  function
> restart; Digits:=15;
```

$$\text{Digits} := 15$$

```
> FT(f,omega) = halfFT(` f`[`+`],omega) + halfFT(` f`[`-`],-omega);
  halfFT(f,omega) = Int(f(x)*exp(I*omega*x),x=0..infinity);
  Int(f(x)*exp(I*omega*x),x=-infinity..0):
  '%'=combine(Change(%,x=-xi,xi));
  #`` = Int(f(-xi)*exp(-2*I*omega*xi)*exp(I*omega*x),x=0..infinity);
```

$$FT(f, \omega) = halfFT(f_{+}, \omega) + halfFT(f_{-}, -\omega)$$

$$halfFT(f, \omega) = \int_{0}^{\infty} f(x)\, e^{(\omega x 1)}\, dx$$

$$\int_{-\infty}^{0} f(x)\, e^{(\omega x 1)}\, dx = \int_{0}^{\infty} f(-\xi)\, e^{(-1 \omega \xi)}\, d\xi$$

```
>
```

complex function as input

```
> # Richtig
  deft_cplx:=proc(real_f, imag_f, omega)
  local h, M,a,b,t,epsMinus,epsPlus,
    nPlus,nMinus,n,
    t01, t02, t03, t04, t05, t06, t07,
    t11,t12,t13,t14,t15,t16,t17,t18,t19,t20,t21,t22,
    s0_tmp, s0_real, s0_imag, sn_tmp, sn_real, sn_imag,
    resultReal,resultImag,
    s_real,s_imag, p;

  h:= 3/40; #0.075;

  # compute lower and upper bound
    epsMinus:=1e-24; epsPlus:=1e-24;
    M := Pi / (omega * h);
    b := 1/4;
    a := b / sqrt(1 + M * ln(1 + M) / (4 * Pi));

    t := ln(-ln(epsPlus / M) / b + 1);
    t := ln((2 * t - ln(epsPlus / M / t)) / b + 1);
    nPlus := round( 1 + floor(ln((2 * t - ln(epsPlus / M / t)) / b + 1) / h)
  );
    t := ln(-ln(epsMinus / M) / a + 1);
    t := ln((2 * t - ln(epsMinus / M / t)) / a + 1);
    nMinus := round( -1 - floor (ln((2 * t - ln(epsMinus / M / t)) / a + 1)
  / h) );

  # compute summand for n=0
    t01 := Pi/h;
    t02 := 1/omega*t01;
    t03 := sqrt(Pi/(4*Pi+ln(t02+1)*t02))/2;
  #1/2*(Pi/(4*Pi+ln(t02+1)*t02))^(1/2);
    t04 := 9/4+t03;
    t05 := 1/t04;
    t06 := t05*t01/2;
    t07 := sin(t06);
    #s0 := 1/2*f(t05*t02)*((t03-1/4)/t04^2+1)*(cos(t06)+t07*I)*t07;
    #  s0_tmp := f(t05*t02)*((t03-1/4)/(t04*t04)+1)*t07/2;
    #  s0_real := s0_tmp*cos(t06);
    #  s0_imag := s0_tmp*t07;
    s0_tmp:=((t03-1/4)/(t04*t04)+1)*t07/2;
    s0_real := s0_tmp*(real_f(t05*t02)*cos(t06) - imag_f(t05*t02)*t07);
    s0_imag := s0_tmp*(real_f(t05*t02)*t07      +
  imag_f(t05*t02)*cos(t06));
    #s0 := s0_real + s0_imag*I;

    s0_real := evalhf(s0_real);
    s0_imag := evalhf(s0_imag);

  # prepare and compute the sum
  t11 := 1/omega;
  t12 := Pi*t11/h;
  t13 := -sqrt(Pi/(4*Pi+ln(1+t12)*t12))/2;

  #s:=0;
  s_real:=0; s_imag:=0;
  n := nMinus;
  while n <= nPlus  do
    if n<> 0 then
      t14 := h*n;
      t15 := -exp(t14)/4;
      t16 := exp(-t14);
      t17 := exp(-2*t14+(1-t16)*t13+t15+1/4);
      t18 := 1-t17;
      t19 := 1/t18;
      t20 := Pi*n*t19;
      t21 := t20/2;
      t22 := sin(t21);
      #sn :=
  f(t11*t20)*(t19+1/t18^2*(-2+t16*t13+t15)*t17*t14)*(cos(t21)*t22+(t22^2-mod
  p(floor(n),2))*I);
      #sn_tmp := f(t11*t20)*(t19+1/(t18*t18)*(-2+t16*t13+t15)*t17*t14);
      #sn_real := sn_tmp*cos(t21)*t22;
      #sn_imag := sn_tmp*(t22*t22-modp(floor(n),2));
      sn_tmp:= (t19+1/(t18*t18)*(-2+t16*t13+t15)*t17*t14);
      sn_real := sn_tmp*( real_f(t11*t20)*cos(t21)*t22               -
  imag_f(t11*t20)*(t22*t22-modp(floor(n),2)) );
      sn_imag := sn_tmp*( real_f(t11*t20)*(t22*t22-modp(floor(n),2)) +
  imag_f(t11*t20)*cos(t21)*t22);
      #sn := sn_real + sn_imag*I;

      #s:= s + evalhf( sn );
      s_real:= evalhf(s_real + sn_real);
      s_imag:= evalhf(s_imag + sn_imag);
    end if;
    n:=n+1;
  end do;

  #s:= s+ s0;
  s_real:= s_real + s0_real;
  s_imag:= s_imag + s0_imag;

  print ( -nMinus+nPlus+1 );
  return evalf( -2*Pi*(s_real*I+s_imag)/omega );
  end proc; # maplemint(%);
```

```
deft_cplx := proc(real_f, imag_f, ω)
local h, M, a, b, t, epsMinus, epsPlus, nPlus, nMinus, n, t01, t02, t03, t04, t05, t06, t07, t11, t12, t13, t14, t15,
t16, t17, t18, t19, t20, t21, t22, s0_tmp, s0_real, s0_imag, sn_tmp, sn_real, sn_imag, resultReal, resultImag,
s_real, s_imag, p;
    h := 3 / 40;
    epsMinus := 0.1*10^(-23);
    epsPlus := 0.1*10^(-23);
```

```
    M := π / (ω*h);
    b := 1 / 4;
    a := b / sqrt(1 + 1 / 4*M*ln(1 + M) / π);
    t := ln(–ln(epsPlus / M) / b + 1);
    t := ln((2*t – ln(epsPlus / (M*t))) / b + 1);
    nPlus := round(1 + floor(ln((2*t – ln(epsPlus / (M*t))) / b + 1) / h));
    t := ln(–ln(epsMinus / M) / a + 1);
    t := ln((2*t – ln(epsMinus / (M*t))) / a + 1);
    nMinus := round(–1 – floor(ln((2*t – ln(epsMinus / (M*t))) / a + 1) / h));
    t01 := π / h;
    t02 := t01 / ω;
    t03 := 1 / 2*sqrt(π / (4*π + ln(t02 + 1)*t02));
    t04 := 9 / 4 + t03;
    t05 := 1 / t04;
    t06 := 1 / 2*t05*t01;
    t07 := sin(t06);
    s0_tmp := 1 / 2*((t03 – 1 / 4) / (t04*t04) + 1)*t07;
    s0_real := s0_tmp*(real_f(t05*t02)*cos(t06) – imag_f(t05*t02)*t07);
    s0_imag := s0_tmp*(real_f(t05*t02)*t07 + imag_f(t05*t02)*cos(t06));
    s0_real := evalhf(s0_real);
    s0_imag := evalhf(s0_imag);
    t11 := 1 / ω;
    t12 := π*t11 / h;
    t13 := –1 / 2*sqrt(π / (4*π + ln(1 + t12)*t12));
    s_real := 0;
    s_imag := 0;
    n := nMinus;
    while n ≤ nPlus do
        if n ≠ 0 then
            t14 := h*n;
            t15 := –1 / 4*exp(t14);
            t16 := exp(–t14);
            t17 := exp(–2*t14 + (1 – t16)*t13 + t15 + 1 / 4);
            t18 := 1 – t17;
            t19 := 1 / t18;
            t20 := π*n*t19;
            t21 := 1 / 2*t20;
            t22 := sin(t21);
            sn_tmp := t19 + 1*(–2 + t16*t13 + t15)*t17*t14 / (t18*t18);
            sn_real := sn_tmp*
                (real_f(t11*t20)*cos(t21)*t22 – imag_f(t11*t20)*(t22*t22 – modp(floor(n), 2)));
            sn_imag := sn_tmp*
                (real_f(t11*t20)*(t22*t22 – modp(floor(n), 2)) + imag_f(t11*t20)*cos(t21)*t22);
            s_real := evalhf(s_real + sn_real);
            s_imag := evalhf(s_imag + sn_imag)
        end if;
        n := n + 1
    end do;
    s_real := s_real + s0_real;
    s_imag := s_imag + s0_imag;
    print(–nMinus + nPlus + 1);
    return evalf(–2*π*(s_real*I + s_imag) / ω)
end proc
> 
> #fTst:=evalf@(D(LambertW));
  fTst:=proc(z) local t1 := evalf(LambertW(z)); return( 1/z/(1+t1)*t1); end
  proc;
  imag_fTst:= z -> 0;
                fTst := proc(z) local t1; t1 := evalf(LambertW(z)); return t1 / (z*(1 + t1)) end proc
                                                    imag_fTst := z → 0
> fTst(2.0);
                                                    0.230109837492918
> wTst:=1.0;
  deft_cplx(fTst, imag_fTst, wTst);

  .323367431677778761399370087952170446651O-.459360912806778795472113334447
  3069368568*I:
  evalf(%);
                                                    wTst := 1.0
                                                deft_cplx(fTst, imag_fTst, 1.0)
                                            0.323367431677779 – 0.459360912806779 I
> 
>
```