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Off-center impact of an elastic column by a rigid mass

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Abstract

Based on the Timoshenko beam model the equations of motion are obtained for large deflection of off-center impact of a column by a rigid mass via Hamilton's principle. These are a set of coupled nonlinear partial differential equations. The Newmark time integration scheme and differential quadrature method are employed to convert the equations into a set of nonlinear algebraic equations for displacement components. The equations are solved numerically and the effects of weight and velocity of the rigid mass and also off-center distance on deformation of the column are studied.

Keywords: Column, Rigid mass, Impact, Off-Center, Contact duration

1. Introduction

The axial impact of a column at the center of cross-section by a rigid mass is an old problem, Davidson (1953). Several investigators analyzed the impact of beams with initial geometric imperfection, see e.g., Hayashi and Sano (1972a, b), Ari-Gur, et al. (1982), Ji and Waas (2013). The results of their analyses were dependent upon the amplitude of the initial imperfection. In the study of the impact of perfect columns conducted by Wang and Tian (2007), and Ji and Waas (2008), at the outset, a non-linear term in a differential equation of motion was ignored which led to the differential equation for linear wave propagation in axial direction. The solution to this equation was, then, used to investigate the transverse motion of the column. The nonlinear vibration of the Euler-Bernoulli beam under transverse periodic load subjected to axial impact was the subject of study by Awrejcewicz et al. (2011). In practice, however, the off-center impact of a straight column by a striking mass may occur, which leads to both transverse and axial deformation of the column. Contrary to the central impact of column by a mass, off-center impact has rarely been addressed in the literature. Kuo (1961), experimentally investigated eccentric longitudinal impact of a horizontally suspended free beam by a striking bar which had a canonical end to exert the impact. A theoretical analysis of the response of the beam using Timoshenko beam theory was also carried out. The governing equations were a set of linear partial differential equations which was solved by the method of characteristics. The oblique impact at the free end of a cantilever column by a rigid mass was the subject of study by Ren (1985). The Timoshenko beam model was employed and the equations of motion were obtained employing Hamilton's principle. These were three non-linear partial differential equations which were solved by finite difference method. The dependencies of impact duration on the incidence angle and also on the slenderness ratio of a column were investigated. In another article, Ren and Kou (1987) conducted an experiment to verify the results of their earlier analysis.

The off-center impact of a column by a rigid mass is the subject of the present study. The Timoshenko beam model is utilized. Therefore, the analysis is valid for moderately thick columns as well as slender ones. The application of Hamilton's principle results in a set of coupled non-linear differential equations for the displacement components of the column. These equations are solved numerically for an impacted column with simply-supported or clamped condition at the fixed end. The results are in excellent agreement with those obtained by finite element method. The effects of velocity, off-center distance and weight of impacting mass on the deformation of column are studied.

2. Formulation

We consider a column with length l, thickness b, and height h. The dimensions are in the $x-$, $y-$, and $z-$directions, respectively. The column is simply-supported at $x = l$; thus, at this end, transverse displacement vanishes. The equations representing displacement field in the Timoshenko beams are

$$u_x = u(x,t) + z\phi(x,t)$$

$$u_z = w(x,t) \qquad (1)$$

where, u and w are, respectively, the axial and transverse deformations of the beam axis, and ϕ, is the rotation of its cross-section. The stationary column is initially impacted by a rigid mass M with a velocity V_0 at the distance $z = e$ from the center of the cross-section. The kinetic energy of the system, in view of Eqs (1), becomes

$$T = \frac{\rho b}{2}\int_0^l \int_{-h/2}^{h/2} \left[\left(\frac{\partial u}{\partial t} + z\frac{\partial \phi}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dx dz + \frac{M}{2}\left[\frac{\partial u}{\partial t}(l,t) + e\frac{\partial \phi}{\partial t}(l,t)\right]^2 \qquad (2)$$

where, ρ is the mass density of the column. The potential energy, under the hypothesis of elastic impact, yields

$$U = \frac{b}{2}\int_0^l \int_{-h/2}^{h/2} (\sigma_{xx}\varepsilon_{xx} + \kappa\sigma_{xz}\gamma_{xz})\, dx dz - Mgu(l,t) - Mge\phi(l,t) \qquad (3)$$

where $\kappa = 5/6$ is the correction shear factor. For isotropic beams with infinitesimal strains the constitutive equations obey Hooke's; thus, $\sigma_{xx} = E\varepsilon_{xx}, \sigma_{xz} = G\gamma_{xz}$, where, E and G are Young's and shear moduli, respectively. The von-Karman strain measure for a Timoshenko beam results in

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \phi \qquad (4)$$

Therefore, potential energy (3) in view of Eqs (4), becomes

$$U = \frac{b}{2}\int_0^l \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{E\left[\frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]^2 + G\kappa\left[\frac{\partial w}{\partial x} + \phi\right]^2\right\} dx dz - Mgu(l,t) - Mge\phi(l,t) \qquad (5)$$

Hamilton's principle states that

$$\int_0^{t_0} \delta(T-U)\, dt = 0 \tag{6}$$

where, $[0, t_0]$ is a small but known time interval. Substituting Eqs (2) and (5) into Eq. (6) and carrying out the required manipulation leads to the following equations of motion

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\kappa}{2(1+\nu)}\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{3}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 = \frac{1}{C^2}\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{6\kappa}{(1+\nu)h^2}\left(\frac{\partial w}{\partial x} + \phi\right) = \frac{1}{C^2}\frac{\partial^2 \phi}{\partial t^2} \tag{7}$$

where, ν is the Poisson's ratio and $C = \sqrt{E/\rho}$ is the longitudinal wave velocity in the material. The above equations are subjected to the following natural and geometric boundary conditions at, $x = l$

$$\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{M}{Ebh}\left(g - \frac{\partial^2 u}{\partial t^2} - e\frac{\partial^2 \phi}{\partial t^2}\right) = 0,$$

$$\frac{\partial \phi}{\partial x} - \frac{12Me}{Ebh^3}\left(g - \frac{\partial^2 u}{\partial t^2} - e\frac{\partial^2 \phi}{\partial t^2}\right) = 0, \qquad w = 0 \tag{8}$$

And at, $x = 0$

$$u = 0, \quad \text{and} \quad w = 0, \tag{9}$$

The last boundary condition at $x = 0$ for a column with simply-supported and clamped boundaries are $\frac{\partial \phi}{\partial x} = 0$, and $\phi = 0$, respectively. Let the impacted column be stationary at $t = 0$, the initial conditions for Eqs (7), yield

$$u(x,0) = \phi(x,0) = w(x,0) = 0$$

$$\frac{\partial \phi}{\partial t}(x,0) = \frac{\partial w}{\partial t}(x,0) = 0, \qquad x \epsilon [0, l]$$

$$\frac{\partial u}{\partial t}(x,0) = 0, \qquad x \epsilon [0, l)$$

$$\frac{\partial u}{\partial t}(l,0) = -V_0 \tag{10}$$

Furthermore, as the axial force at $x = l$ vanishes, the impacting mass losses contact with the column. From first Eq. (4) and Hooke's law, we arrive at the equation of the contact duration, t_c, as

$$\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]_{x=l} = 0 \tag{11}$$

The Newmark and differential quadrature methods are used to solve the system of non-linear equations (7). The application of these methods results in

$$\sum_{j=2}^{N} c_{ij}^{(2)} u^{k+1}(x_j) + \left(\sum_{j=2}^{N-1} c_{ij}^{(1)} w^{k+1}(x_j)\right)\left(\sum_{j=2}^{N-1} c_{ij}^{(2)} w^{k+1}(x_j)\right) - \frac{1}{C^2 \Delta t^2} u^{k+1}(x_i)$$

#

$$= -\frac{1}{C^2}\left(\frac{1}{\Delta t^2} u^k(x_i) + \frac{1}{\Delta t}\dot{u}^k(x_i) - \frac{1}{2}\ddot{u}^k(x_i)\right),$$

$$\frac{5}{12(1+\nu)}\left(\sum_{j=m}^{N} c_{ij}^{(1)} \phi^{k+1}(x_j) + \sum_{j=2}^{N-1} c_{ij}^{(2)} w^{k+1}(x_j)\right)$$

#

$$+\left(\sum_{j=2}^{N} c_{ij}^{(2)} u^{k+1}(x_j)\right)\left(\sum_{j=2}^{N-1} c_{ij}^{(1)} w^{k+1}(x_j)\right) + \left(\sum_{j=2}^{N} c_{ij}^{(1)} u^{k+1}(x_j)\right)\left(\sum_{j=2}^{N-1} c_{ij}^{(2)} w^{k+1}(x_j)\right)$$

#

$$+\frac{3}{2}\left(\sum_{j=2}^{N-1} c_{ij}^{(2)} w^{k+1}(x_j)\right)\left(\sum_{j=2}^{N-1} c_{ij}^{(1)} w^{k+1}(x_j)\right)^2 - \frac{1}{C^2 \Delta t^2} w^{k+1}(x_i)$$

#

$$= -\frac{1}{C^2}\left(\frac{1}{\Delta t^2} w^k(x_i) + \frac{1}{\Delta t}\dot{w}^k(x_i) - \frac{1}{2}\ddot{w}^k(x_i)\right),$$

$$\sum_{j=m}^{N} c_{ij}^{(2)}\,\phi^{k+1}(x_j) - \frac{5}{(1+\nu)h^2}\sum_{j=2}^{N-1} c_{ij}^{(1)}\,w^{k+1}(x_j) - \left(\frac{1}{C^2\Delta t^2}+\frac{5}{(1+\nu)h^2}\right)\phi^{k+1}(x_i)$$
#
$$= -\frac{1}{C^2}\left(\frac{1}{\Delta t^2}\phi^k(x_i)+\frac{1}{\Delta t}\dot{\phi}^k(x_i)-\frac{1}{2}\ddot{\phi}^k(x_i)\right), \qquad i = 2, \ldots, N-1 \tag{12}$$

Analogously, boundary conditions (8) may be written as

$$\sum_{j=2}^{N} c_{Nj}^{(1)}\,u^{k+1}(x_j) + \frac{1}{2}\left(\sum_{j=2}^{N-1} c_{Nj}^{(1)}\,w^{k+1}(x_j)\right)^2 + \frac{M}{Ebh\Delta t^2}\left(u^{k+1}(x_N)+e\phi^{k+1}(x_N)\right)$$
#
$$= \frac{M}{Ebh}\left[g+\frac{1}{\Delta t^2}\left(u^k(x_N)+e\phi^k(x_N)\right)+\frac{1}{\Delta t}\left(\dot{u}^k(x_N)+e\dot{\phi}^k(x_N)\right)-\frac{1}{2}\left(\ddot{u}^k(x_N)+e\ddot{\phi}^k(x_N)\right)\right],$$

$$\sum_{j=m}^{N} c_{Nj}^{(1)}\,\phi^{k+1}(x_j) + \frac{12Me}{Ebh^3\Delta t^2}\left(u^{k+1}(x_N)+e\phi^{k+1}(x_N)\right)$$
#
$$= \frac{12Me}{Ebh^3}\left[g+\frac{1}{\Delta t^2}\left(u^k(x_N)+e\phi^k(x_N)\right)+\frac{1}{\Delta t}\left(\dot{u}^k(x_N)+e\dot{\phi}^k(x_N)\right)-\frac{1}{2}\left(\ddot{u}^k(x_N)+e\ddot{\phi}^k(x_N)\right)\right],$$

$$\sum_{j=m}^{N} c_{1j}^{(1)}\,\phi^{k+1}(x_j) = 0 \qquad only\ for\ a\ beam\ simply-supported\ at\ x=0 \tag{13}$$

The value of m for a beam simply-supported (clamped) at $x = 0$ is $m = 1$ ($m = 2$). Moreover, the initial conditions, $(k = 0)$, are

$$u^0 = w^0 = \phi^0 = \dot{w}^0 = \dot{\phi}^0 = \ddot{w}^0 = \ddot{\phi}^0 = 0 \quad for \quad x\epsilon[0,l]$$

$$\dot{u}^0 = \ddot{u}^0 = 0 \quad for \quad x\epsilon[0,l)$$

$$\dot{u}^0 = -V_0 \quad for \quad x = l$$

$$\ddot{u}^0 = g \quad for \quad x = l \tag{14}$$

whereas, for, $t > 0$, $(k \geq 1)$, the acceleration and velocity at a point are

$$\psi^{k+1} = \frac{1}{\Delta t^2}(\psi^{k+1} - \psi^k) - \frac{1}{\Delta t}\psi^k + \frac{1}{2}\psi^k$$

$$\psi^{k+1} = \frac{3}{2\Delta t}(\psi^{k+1} - \psi^k) - \frac{1}{2}\psi^k + \frac{\Delta t}{4}\psi^k, \qquad \Psi \in \{u, \phi, w\} \tag{15}$$

In Eqs (12), N is the number of sampling points on the column, $0 < x < l$, and cosine distribution is employed to define the location of sampling points

$$x_i = \frac{l}{2}\left[1 - cos\left(\frac{i-1}{N-1}\pi\right)\right] \qquad i = 1,2,\dots,N \tag{16}$$

The weighting coefficients $c_{ij}^{(n)}$ for the determination of nth order derivative are, see for instance Chang (2000)

$$c_{ij}^{(1)} = \frac{H(x_i)}{(x_i - x_j)H(x_j)} \quad i,j = 1,2,\dots,N \; and \; i \neq j, \qquad c_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} c_{ij}^{(1)}$$

$$c_{ij}^{(n)} = n\left(c_{ij}^{(1)} c_{ii}^{(n-1)} - \frac{c_{ij}^{(n-1)}}{x_i - x_j}\right) \quad i,j = 1,2,\dots,N \; and \; i \neq j, \qquad c_{ii}^{(n)} = -\sum_{j=1, j\neq i}^{N} c_{ij}^{(n)} \tag{17}$$

where, the polynomial $H(x_k)$ is

$$H(x_k) = \prod_{j=1, j\neq k}^{N} (x_k - x_j)$$

The systems of non-linear algebraic equations (12) and (13) are solved in each time-step Δt by means of the Newton-Raphson method to evaluate displacement components.

3. Numerical results

We consider a column made up of steel (ASTM-A693 Grade 630) with the yield stress, $\sigma_y = 1170(MPa)$. Unless otherwise stated, the following numerical values are used in the study

$$l = 25(cm), \qquad b = 1(cm), \qquad h = 0.4(cm), \qquad \rho = 7900(kg/m^3),$$

$$E = 200(GPa), \quad \nu = 0.3, \quad M = 0.5(kg), \quad V_0 = 5(m/s), \quad e = 0.1(cm) \tag{18}$$

It is worth mentioning that the numerical values are such that the column may not collapse under plastic deformation. The convergence of numerical results, for the worst case which is impact with the highest velocity, is achieved by choosing the time-increment $\Delta t = 0.1\text{x}10^{-3}(ms)$ and number of sampling points $N = 101$. The duration of contact between the rigid mass and the column simply-supported at $x = 0$ is $t_c = 0.375(ms)$ whereas for the clamped column $t_c = 0.3786(ms)$. We should also note that the analysis is valid for $t \leq t_c$. The normalized transverse deflection $w(x, t_c)/h$ is depicted in Figs 1 and 2. Moreover, the normalized axial displacement, $u(l, t)/h$ verses normalized time tC/l is displayed in Figs 3 and 4. At time t_c the axial displacement at $x = l$ does not vanish; thus columns are still under compression. Another analysis of the problem is carried out using ABAQUS/Explicit finite element software. The finite element mesh comprised of two-hundred, C3D8R elements. Close correlation between the results of two procedures justifies the accuracy of the present analysis. By comparing Figs 1and 3 with 2, and 4, respectively, we may observe that the effect of boundary conditions at $x = 0$ is quite local. Therefore, in the following, only simply-supported columns at the fixed end are considered.

As another example, we study the effect of weight of impacting mass on the transverse deformation of the column, Fig. 5. The time t_c and the maxima of transverse deflection, the latter as time progresses, increase noticeably for heavier impacting masses.

The axial strain of the column at $x = l$ and $z = 0$ versus normalized time for different values of off-center distance e is shown in Fig. 6. For $e = 0$ only axial displacement of a perfect column occurs. It can be observed that the time t_c does not change with off-center distance. Besides, the increase of off-center distance results in higher bending moment in the column. Therefore, from Eq. (4), we deduce that the magnitude of $\varepsilon_{xx}(l, 0, t)$ decreases with increasing off-center distance.

The last example, deals with the column which is impacted by the rigid mass with different velocities, V_0, Fig. 7. The variation of time t_c with the impact velocity is negligible. This is in accord with a finding in, Ji and Waas (2008). However, the transverse deflection of the column magnifies significantly by the increase of impact velocity.

4. Conclusion

We have analyzed the off-center impact of an elastic perfect column by a rigid mass in the course of the first impact. The von-Karman strain measure is employed; thus analysis is valid for columns undergoing large deflection. The effect of boundary conditions at the fixed support is local. Therefore, the overall behavior of columns clamped or simply-supported, at the fixed end, is similar. The effect of impact velocity on the duration of impact is negligible but it is drastic on the transverse deformation of the column. However, as the weight of impacting mass increases, the duration of impact and also deflection of the column increase significantly. Moreover, numerical results reveal that duration of contact is the same for different values of off-center distance e and columns remain under compression in the course of impact.

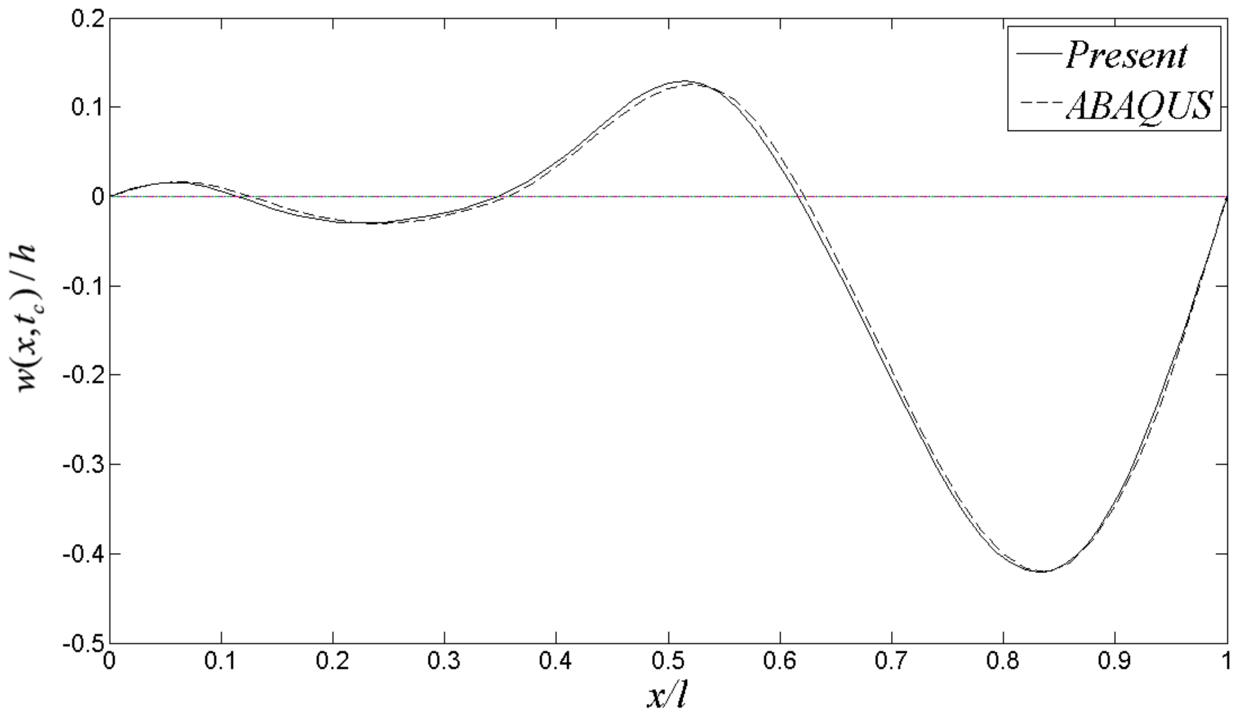

Fig. 1. Transverse deflection of the column simply-supported at x=0

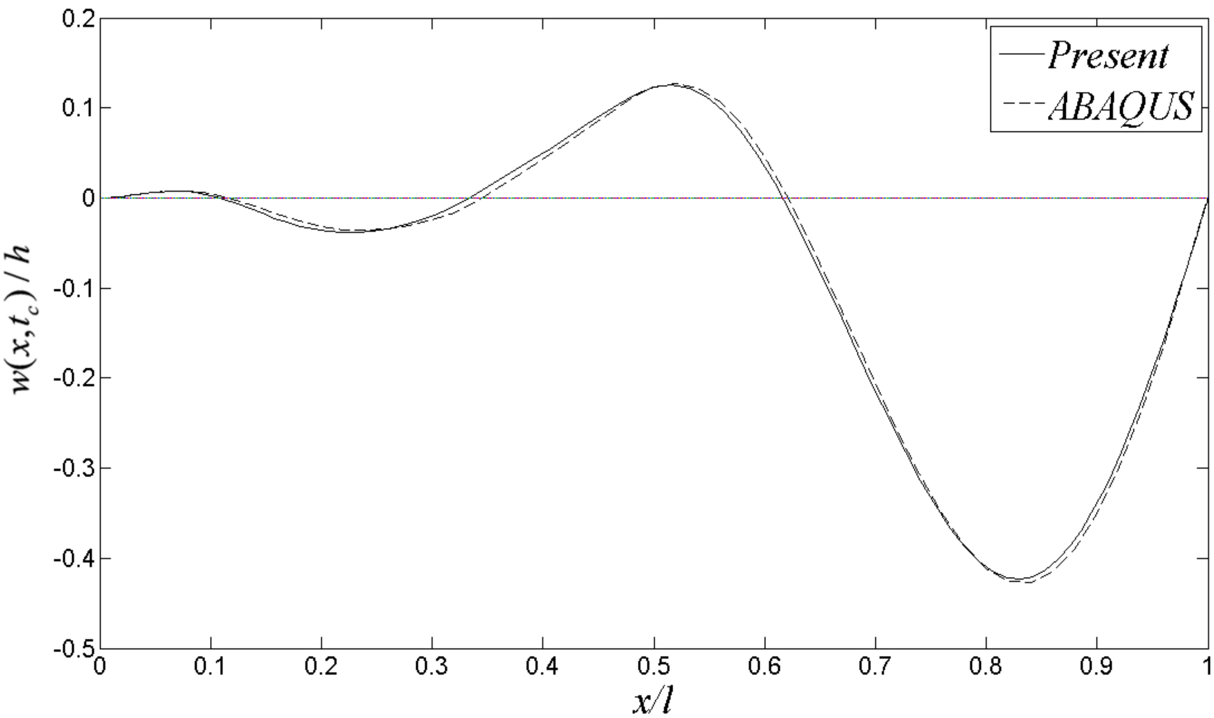

Fig. 2. Transverse deflection of the column clamped at x=0

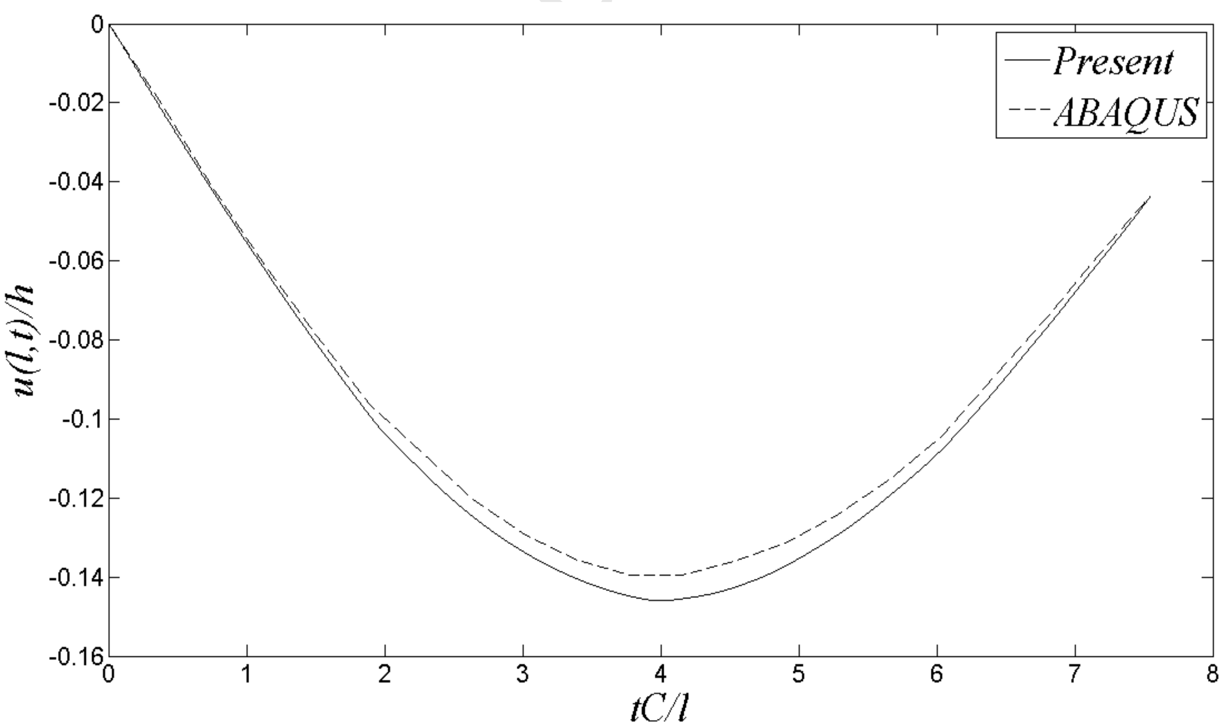

Fig. 3. Axial displacement at the end of the column simply-supported at x=0

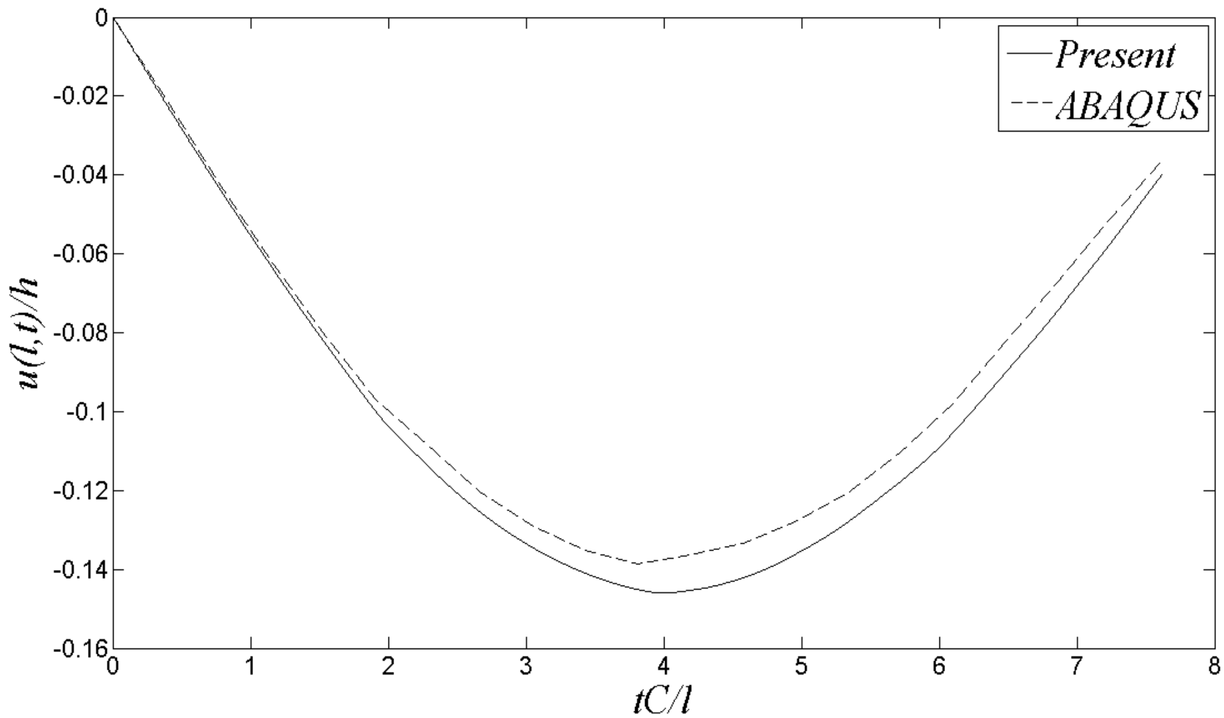

Fig. 4. Axial displacement at the end of the column clamped at x=0

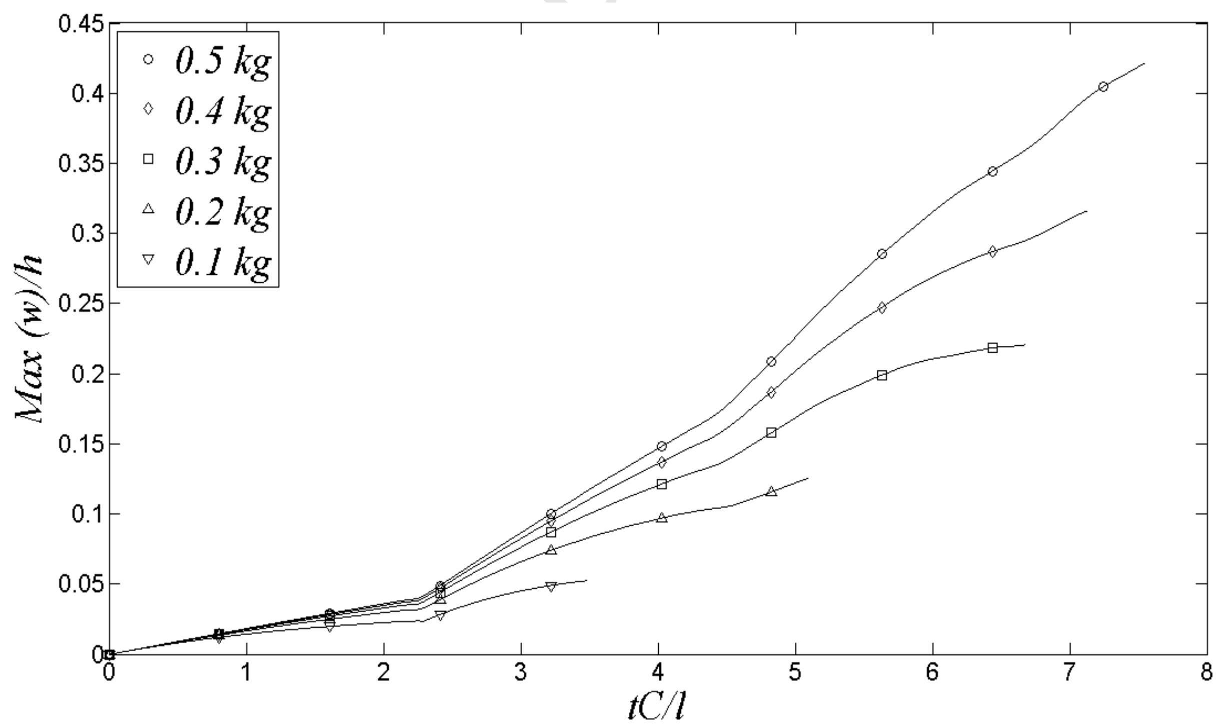

Fig. 5. Maximum transverse deflection of the column with different rigid mass

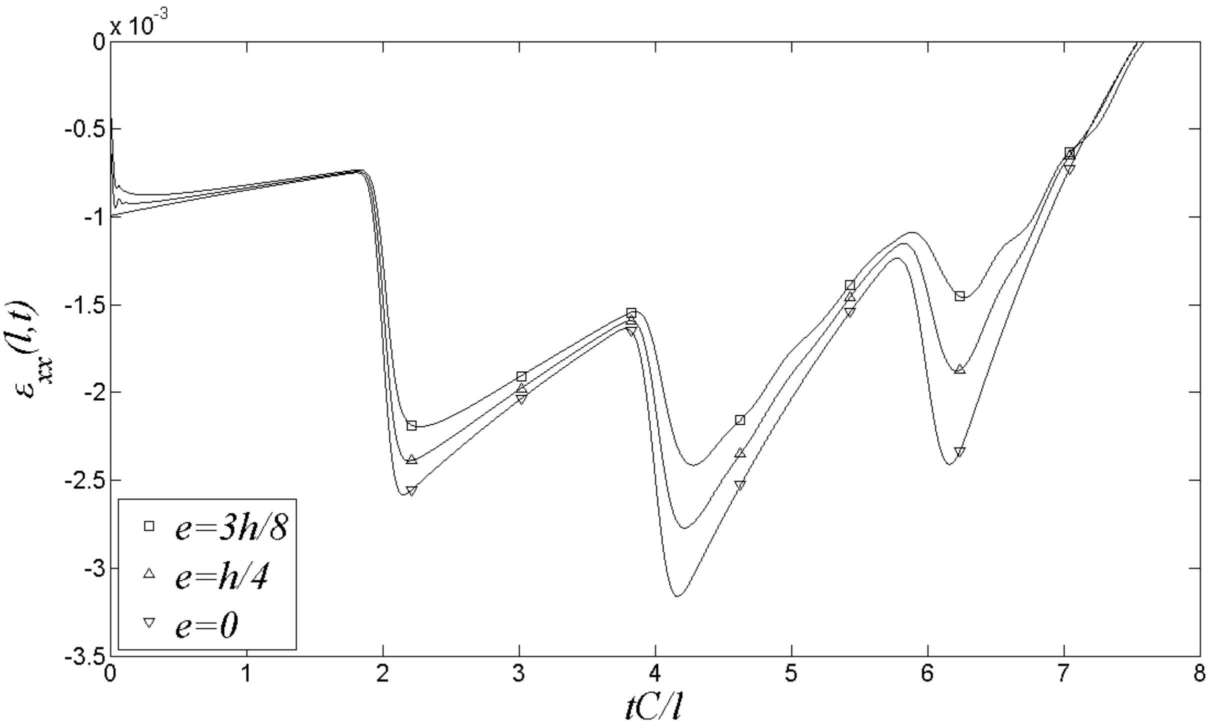

Fig. 6. Axial strain at the contact surface of column for different off-center distances

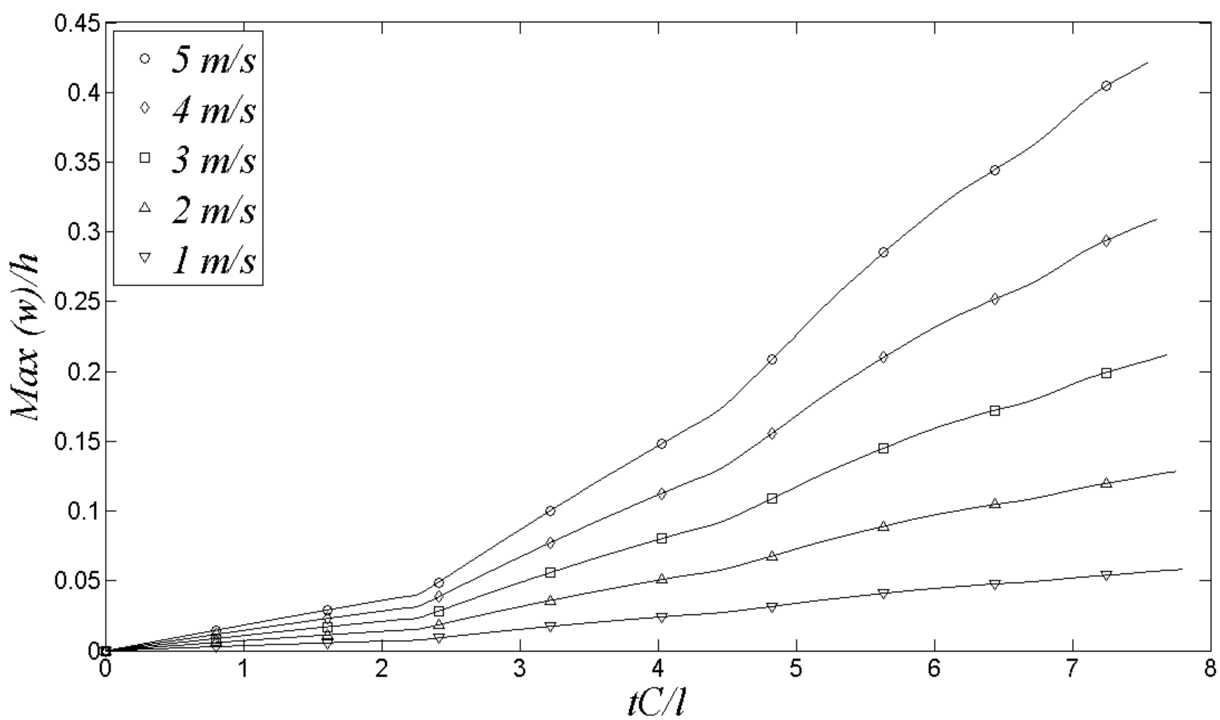

Fig. 7. Effect of mass velocity on transverse deformation of the column

References

Ari-Gur, J., Weller, T., Singer, J., 1982. Experimental and theoretical studies of columns under axial impact, Int. J. Solids Struc. 18(7), 619-641.

Awrejcewicz, J., Krysko, V.A., Saltykova, O.A., Chebotyrevskiy, Yu.B., 2011. Nonlinear vibrations of the Euler-Bernoulli beam subject to transversal load and impact actions, Nonlinear Studies. 18(3), 329-364.

Chang, S., 2000. Differential Quadrature and Its Application in Engineering, Springer, London.

Davidson, J.F., 1953. Buckling of struts under dynamic loading, J. Mech. Phys. Solids 2, 54-66.

Hayashi, T., Sano, Y., 1972. Dynamic buckling of elastic bars, 1st Report: The case of low velocity impact, Bull. JSME 15(88), 1167-1175.

Hayashi, T., Sano, Y., 1972. Dynamic buckling of elastic bars, 2nd Report: The case of High velocity impact, Bull. JSME 15(88), 1176-1184.

Ji, W., Waas, A.M., 2008. Dynamic bifurcation buckling of an impacted column, Int. J. Eng. Sci. 46, 958-967.

Ji, W., Waas, A.M., 2013. The temporal evolution of buckling in a dynamically impacted column, J. Appl. Mech. (ASME) 80, 011026-1/7.

Kuo, S. S., 1961. Beam subjected to eccentric longitudinal impact, Exp. Mech. 1(9), 102-108.

Ren, L.X., 1985. Almost-axial impact on an elastic cantilever column- a theoretical study, J. Sound Vib. 100(3), 321-337.

Ren, L.X., Kou, S.Q., 1987. Slightly slanting impact on an elastic cantilever column-an experimental study, J. Sound Vib. 112(1), 1-14.

Wang, A., Tian, W., 2007. Mechanism of buckling development in elastic bars subjected to axial impact, Int. J. Impact Eng. 34, 232-252.

Highlights

- The governing equations for the off-center impact of an elastic column by a rigid mass are derived.
- The duration of contact between the column and impacting mass is obtained.
- The effects of velocity, weight of impacting mass and off-center distance are studied.