polynomial is in accordance with the determinant of the univariate coefficient matrix, however, many prevenient methods are not so. The algorithm is comparatively concise and requires fairly less computation time enough to be used for real-time applications. A numerical example is given to verify the algorithm and its results without extraneous roots agree with the original equations.

The rest of the paper is organized as follows. In Section II, we give the kinematic constraint equations. In Section Ⅲ, we present the elimination process for solving the kinematic constraint equations. In Section Ⅳ, we give a numerical example verifies the algorithm presented in this paper. In Section Ⅴ, conclusions are given.

II. THE KINEMATIC CONSTRAINT EQUATIONS

Fig. 1 shows the geometric model of the 6-6 Stewart platform with planar base and moving platform. All the joints of its base and moving platform are located in respective planes. The six inputs necessary to describe the location and orientation of the upper platform are the leg lengths controlled by each prismatic joint. For a general case, the absolute local frame system $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$ and the relative moving frame system $\boldsymbol{O}_2$-$\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$ are fixed to the arbitrary points $\boldsymbol{O}_1$ on the base platform and $\boldsymbol{O}_2$ on the moving platform, respectively. The direct kinematics problem is to find the position and orientation of the moving platform supposing that the pose of the base platform is known and values for the six constraints connecting to the base and the platform are given.

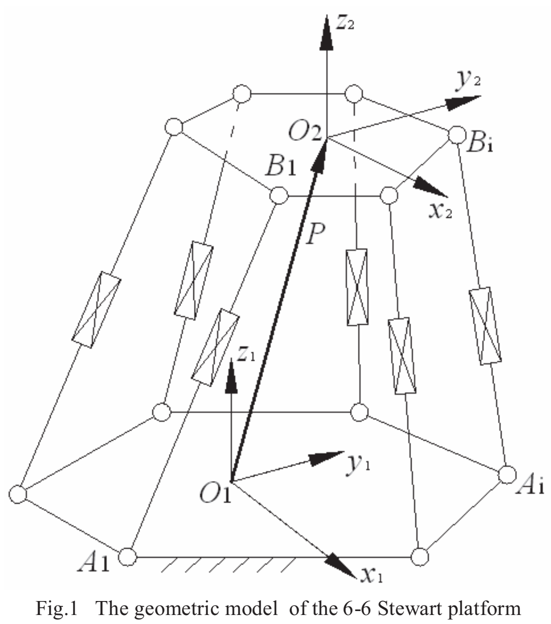

Fig.1 The geometric model of the 6-6 Stewart platform

Let the coordinates of point A_i is (x_i, y_i, 0) (i = 1, 2, 3, 4, 5, 6) in the absolute frame $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$, the coordinates of point B_i is (p_i, q_i, 0) (i = 1, 2, 3, 4, 5, 6) in the moving frame $\boldsymbol{O}_2$-$\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$, and the coordinates of origin point $\boldsymbol{O}_2$ in the absolute frame $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$ is (x_i, y_i, 0). The lengths of $\boldsymbol{A}_i\boldsymbol{B}_i$ are denoted as L_i. Given the position vector $\boldsymbol{P}$ between the two origin points $\boldsymbol{O}_1$ and $\boldsymbol{O}_2$, and the transformation matrix R between the two coordinate systems, the leg vectors can be easily represented as

$$\boldsymbol{L}_i = \boldsymbol{R}\boldsymbol{B}_i + \boldsymbol{P} - \boldsymbol{A}_i,\quad i = 1, 2, 3, 4, 5, 6 \tag{1}$$

Where

$$\boldsymbol{R} = \begin{bmatrix} r_1 & r_4 & r_7 \\ r_2 & r_5 & r_8 \\ r_3 & r_6 & r_9 \end{bmatrix} \tag{2}$$

With given leg lengths, the kinematic constraint equations corresponding to the conditions of constraint length of each leg are as follows

$$(\boldsymbol{P} + \boldsymbol{R}\boldsymbol{B}_i - \boldsymbol{A}_i)^T(\boldsymbol{P} + \boldsymbol{R}\boldsymbol{B}_i - \boldsymbol{A}_i) = L_i^2 \quad i=1, 2, 3, 4, 5, 6 \tag{3}$$

Substituting all coordinates above and (2) into (3), we get

$$(p_i r_1 + q_i r_4 + x - x_i)^2 + (p_i r_2 + q_i r_5 + y - y_i)^2 + (p_i r_3 + q_i r_6 + z)^2 - L_i^2 = 0 \quad i=1,2,3,4,5,6 \tag{4}$$

Since $\boldsymbol{R}$ is orthogonal, r_i (i = 1, …, 9) satisfy the following relations

$$r_1^2 + r_2^2 + r_3^2 - 1 = 0 \tag{5}$$

$$r_4^2 + r_5^2 + r_6^2 - 1 = 0 \tag{6}$$

$$r_1 r_4 + r_2 r_5 + r_3 r_6 = 0 \tag{7}$$

$$r_4 r_8 - r_5 r_7 - r_3 = 0 \tag{8}$$

$$r_2 r_7 - r_1 r_8 - r_6 = 0 \tag{9}$$

$$r_1 r_5 - r_2 r_4 - r_9 = 0 \tag{10}$$

Equations (4), (5), (6), (7) are devoid of unknown variables r_7, r_8, and r_9. If it is necessary, the unknown variables r_7, r_8, and r_9 will be obtained by (8), (9), (10) when other variables are known. So (4), (5), (6), (7), which represent 9 equations in 9 unknowns r_1, r_2, r_3, r_4, r_5, r_6, x, y, z, are the kinematic constraint equations describing the direct kinematics of the 6-6 Stewart platform.

Ⅲ. THE ELIMINATION PROCESS

A. Intermediate Polynomials in Three Variables

Equation (4) can be reduced to

$$p_i x_i r_1 + p_i y_i r_2 - p_i u + q_i x_i r_4 + q_i y_i r_5 - q_i v + x_i x + y_i y - w/2 + m_i = 0 \quad i = 1, 2, 3, 4, 5, 6 \tag{11}$$

where

$$m_i = (L_i^2 - x_i^2 - y_i^2 - p_i^2 - q_i^2)/2$$

$$u = r_1 x + r_2 y + r_3 z \tag{12}$$

$$v = r_4 x + r_5 y + r_6 z \tag{13}$$

$$w = x^2 + y^2 + z^2 \tag{14}$$

Equations (11) are linear with respect to r_1, r_2, r_4, r_5, u, v, x, y, and w, and can be arranged as follows

$$\boldsymbol{M}_{6\times10}\boldsymbol{t} = 0$$

where the i row of the matrix $\boldsymbol{M}_{6\times10}$ is