



Nanofluid two phase model analysis in existence of induced magnetic field



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ABSTRACT

Two phase nanofluid double diffusion convection in presence of induced magnetic field is analyzed. Runge-Kutta method was applied to find the solution of this problem. Numerical procedures are examined for various active parameters namely; Prandtl and Hartmann numbers, suction parameter, Buoyancy ratio, Schmidt number, thermophoretic and Brownian motion parameters. Results revealed that temperature gradient enhances with augment of suction parameter but it reduces with augment of thermophoretic parameters. Nanofluid motion reduces with augment of Schmidt and Hartmann numbers but it enhances with augment of Buoyancy ratio and thermophoretic parameters.

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Induced magnetic field
Current density
Buoyancy ratio
Permeable plate

1. Introduction

Free convection in existence of Lorentz forces has several uses such as geophysics, fire engineering and etc. Newly nanotechnology was suggested as new passive way for heat transfer improvement. Influence of induced magnetic field on temperature curves has been illustrated by Ghosh et al. [1]. Transient flow in presence of inclined magnetic field over a cone has been analyzed by Vanita and Kumar [2]. Impact of Lorentz forces on boundary layer flow has been examined by Beg et al. [3]. Peristaltic magnetic nanofluid flow in a duct has been analyzed by Akbar et al. [4]. The impact of atherosclerosis on hemodynamics of stenosis was predicted by Nadeem and Ijaz [5]. They proved that the velocity gradient reduces with rise of Strommers number. Nanofluid convection in 3D cavity has been presented by Sheikholeslami and Ellahi [6]. They concluded that convection reduces with rise of Hartmann number. Sheremet et al. [7] analyzed the impact of corner heater on nanofluid motion in porous media in existence of magnetic field. Bhatti and Rashidi [8] analyzed Williamson nanofluid over a plate. Impact of nonlinear radiative heat transfer has been presented by Hayat et al. [9].

Sheikholeslami et al. [10] simulated the impact of radiation on nanofluid heat transfer in existence of Lorentz forces. Their outputs indicated that rate of heat transfer reduces with augment of Lorentz forces. Bondareva et al. [11] used heatline method to modeling the MHD free convection in an open cavity. Sheikholeslami et al. [12] presented the $\text{Fe}_3\text{O}_4\text{-H}_2\text{O}$ nanofluid forced convection in curved cavity. They concluded that temperature gradient augments with decrease of Hartmann number. MHD nanofluid free convective hydrothermal analysis in a tilted wavy enclosure was presented by Sheremet et al. [13]. Their results illustrated that change of inclined angle causes convective heat transfer to augment. Influence of non-uniform Lorentz forces on nanofluid flow style was presented by Sheikholeslami Kandalousi [14]. He proved that augmentation in heat transfer reduces with augment of Kelvin forces. Impact of Coulomb forces on ferrofluid convection has been reported by Sheikholeslami and Chamkha [15]. Their results proved that adding electric field has more benefit in low Reynolds number. Thixotropic nanoparticles have been selected by Hayat et al. [16] to investigate heat transfer augmentation. Sheikholeslami and Chamkha [17] studied MHD $\text{Fe}_3\text{O}_4\text{-water}$ flow in a wavy enclosure with moving wall. Sheikholeslami et al. [18] examined the impact of Lorentz forces on forced convection. Their outputs illustrated that greater lid velocity has more sensible Kelvin forces impact. Recently, several researcher published articles about heat transfer [19–32].

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Nomenclature

B	dimensionless induced horizontal magnetic field
b	magnetic field vector
C	concentration of nanofluid
D_T	thermophoretic diffusion coefficient
D_B	Brownian diffusion coefficient
Ha	Hartmann number
J	induced current density
Nt	thermophoretic parameter
Nb	Brownian motion parameter
Pm	magnetic Prandtl number
Pr	Prandtl number
T	temperature
U	dimensionless horizontal velocity
\vec{v}	velocity vector
V_0	suction parameter

Greek symbols	
ϕ	dimensionless concentration
σ	electrical conductivity
ρ	density
β	coefficient of thermal expansion
θ	dimensionless temperature
μ	dynamic viscosity of nanofluid

Subscripts	
p	solid
f	base fluid
c	concentration

The final purpose of this paper is to investigate nanofluid heat and mass transfer considering of induced magnetic field. Runge-Kutta method is used in this study. The influences of Buoyancy ratio, magnetic Prandtl and Hartmann numbers, thermophoretic, suction parameter, Schmidt number and Brownian motion parameters on concentration, temperature, induced magnetic, velocity and current density profiles are presented.

2. Governing equations

Nanofluid fluid through two vertical porous sheets is studied as illustrated in Fig. 1. The boundary conditions are defined in this figure. The variables are only function of y because plates are infinity.

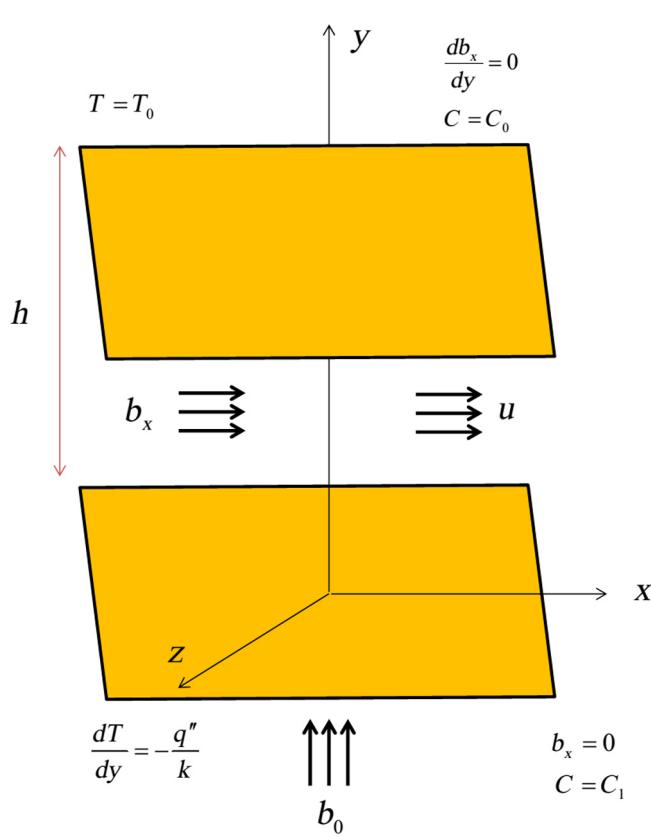


Fig. 1. Geometry of the problem.

Velocity and magnetic field vectors are considered as $\vec{v} = [u, v_0, 0]$ and $\vec{b} = [b_x, b_0, 0]$ respectively. The governing equations and boundary conditions can be obtained as follow:

$$\frac{\mu}{\rho_f} \frac{d^2 u}{dy^2} + \frac{\mu_e b_0}{\rho_f} \frac{db_x}{dy} + g\beta(T - T_0) + g\beta_c(C - C_0) + v_0 \frac{du}{dy} = 0 \quad (1)$$

$$\frac{1}{\mu_e \sigma} \frac{d^2 b_x}{dy^2} + b_0 \frac{du}{dy} + v_0 \frac{db_x}{dy} = 0 \quad (2)$$

$$\alpha \frac{d^2 T}{dy^2} + v_0 \frac{dT}{dy} + \frac{(\rho c)_p}{(\rho c)_f} \left[D_B \left\{ \frac{dC}{dy} \cdot \frac{dT}{dy} \right\} + (D_T/T_0) \left\{ \left(\frac{dT}{dy} \right)^2 \right\} \right] = 0 \quad (3)$$

$$v_0 \frac{d\phi}{dy} + D_B \left\{ \frac{d^2 C}{dy^2} \right\} + \left(\frac{D_T}{T_0} \right) \left\{ \frac{d^2 C}{dy^2} \right\} = 0 \quad (4)$$

$$C = C_1, u = 0, b_x = 0, \frac{dT}{dy} = -\frac{q}{k}, \text{ at } y = 0 \quad (5)$$

$$C = C_0, u = 0, \frac{db_x}{dy} = 0, T = T_0 \text{ at } y = h \quad (6)$$

Dimension less parameters are presented as:

$$\begin{aligned} Y &= \frac{y}{h}, U = \frac{\mu u}{\rho_f g \beta h^2 \Delta T}, B = \sqrt{\frac{\mu_e}{\rho_f}} b_x \frac{\mu}{\beta h^2 \rho_f g \Delta T}, \\ \theta &= \Delta T^{-1}[T - T_0], \Delta T = \frac{hq}{k}, \theta = \frac{C - C_0}{\Delta C}, \Delta C = C_1 - C_0, \\ \text{Pr} &= \frac{\mu}{\rho_f \alpha}, Pm = \frac{\mu}{\rho_f} \sigma \mu_e, Ha = \frac{B_0 h}{v} \sqrt{\frac{\mu_e}{\rho}}, V_0 = \frac{v_0 h}{v}, Sc = \frac{\mu}{\rho_f D}, \\ Br &= \frac{\beta_c \Delta C}{\beta \Delta T}, Nb = \frac{(\rho C_p)_p D_B \Delta C}{(\rho C_p)_f \alpha}, Nt = \frac{(\rho C_p)_p D_T \Delta T}{(\rho C_p)_f \alpha T_0} \end{aligned} \quad (7)$$

Table 1

Comparison of skin friction coefficient over the upper wall between the present results and previous work.

V_0	Pr	Pr_m	Ha	Sarveshanand and Singh [33]	Present work
0.5	0.7	0.5	5	0.016061	0.015996
0.75	0.7	0.5	5	0.010222	0.010853
1	0.7	1	5	0.018209	0.018113
1	0.015	0.5	0.5	2.69599	2.700112

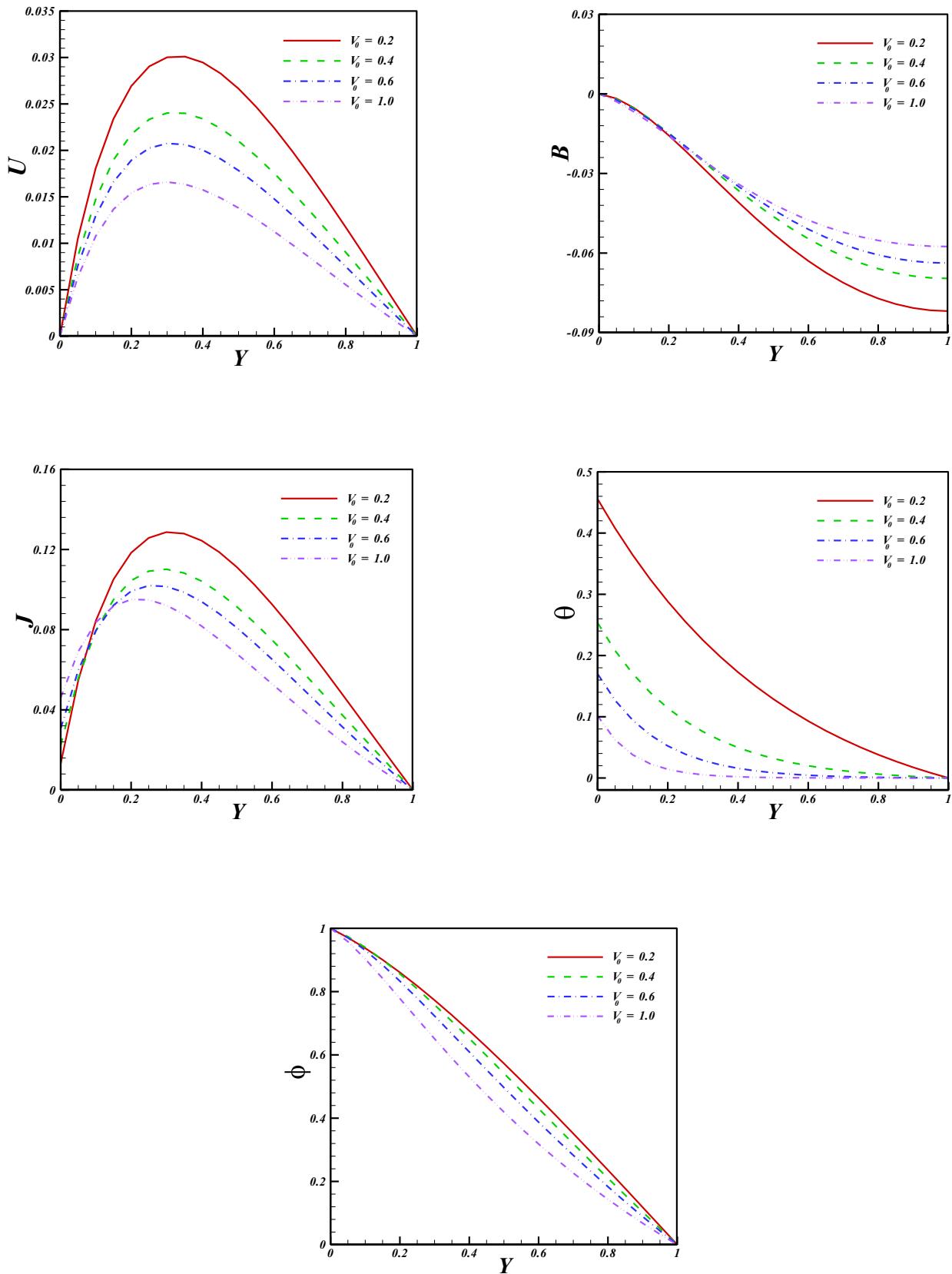


Fig. 2. Effect of suction parameter on velocity, induced magnetic field, induced current density, temperature and concentration distributions $Ha = 5$, $Pm = 0.8$, $Nt = 0.1$, $Nb = 0.1$, $Br = 1$, $Sc = 1$, $Pr = 10$.

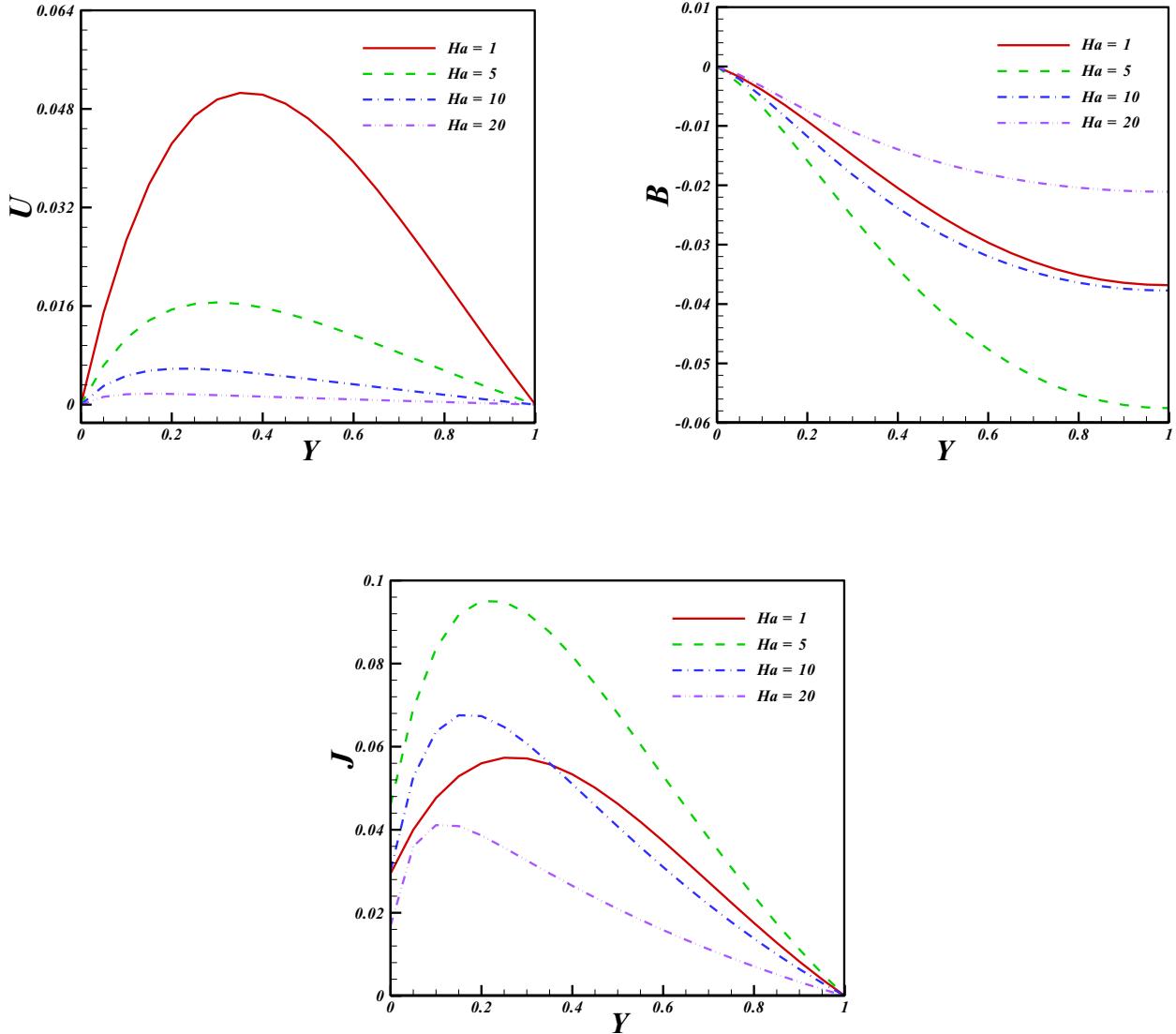


Fig. 3. Effect of Hartmann number on velocity, induced magnetic field and induced current density distributions when $V_0 = 1$, $Pm = 0.8$, $Nt = 0.1$, $Nb = 0.1$, $Br = 1$, $Sc = 1$, $Pr = 10$.

Finally, the dimension less governing equations are:

$$\frac{d^2U}{dY^2} + Ha \frac{dB}{dY} + \theta + Br\phi + V_0 \frac{dU}{dY} = 0 \quad (8)$$

$$\frac{d^2B}{dY^2} + V_0 Pm \frac{dB}{dY} + Ha Pm \frac{dU}{dY} = 0 \quad (9)$$

$$\frac{d^2\theta}{dY^2} + Nt \left(\frac{d\theta}{dY} \right)^2 + V_0 Pr \frac{d\theta}{dY} + Nb \frac{d\theta}{dY} \frac{d\phi}{dY} = 0 \quad (10)$$

$$\frac{d^2\phi}{dY^2} + \frac{Nt}{Nb} \frac{d^2\theta}{dY^2} + V_0 Sc \frac{d\phi}{dY} = 0 \quad (11)$$

$$\phi = 1, B = 0, U = 0, \frac{d\theta}{dy} = -1, \text{ at } Y = 0 \quad (12)$$

$$\phi = 0, \frac{dB}{dY} = 0, U = 0, \theta = 0, \text{ at } Y = 1 \quad (13)$$

Induced current density can be defined:

$$J = -\frac{dB}{dY} \quad (13)$$

C_f and Nu can be expressed as:

$$C_f = U'(0), \quad Nu = 1/\theta(0). \quad (14)$$

3. Runge-Kutta method

In Runge-Kutta, at first the following definitions are applied: $x_2 = U$, $x_1 = Y$, $x_3 = U'$, $x_4 = B$, $x_5 = B'$, $x_8 = \phi$, $x_9 = \phi'$, $x_6 = \theta$, $x_7 = \theta'$. The final system and initial conditions are:

$$\begin{pmatrix} 1 \\ x_3 \\ -x_6 - Ha x_5 - V_0 x_3 - Br x_8 \\ x_5 \\ -(Ha x_3 + V_0 x_5) Pm \\ x_7 \\ -Nt (x_7)^2 - V_0 x_7 Pr - Nb x_7 x_9 \\ x_9 \\ -V_0 Sc x_9 - \frac{Nt}{Nb} x_7' \end{pmatrix} = \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \\ x'_4 \\ x'_5 \\ x'_6 \\ x'_7 \\ x'_8 \\ x'_9 \end{pmatrix} \quad (15)$$

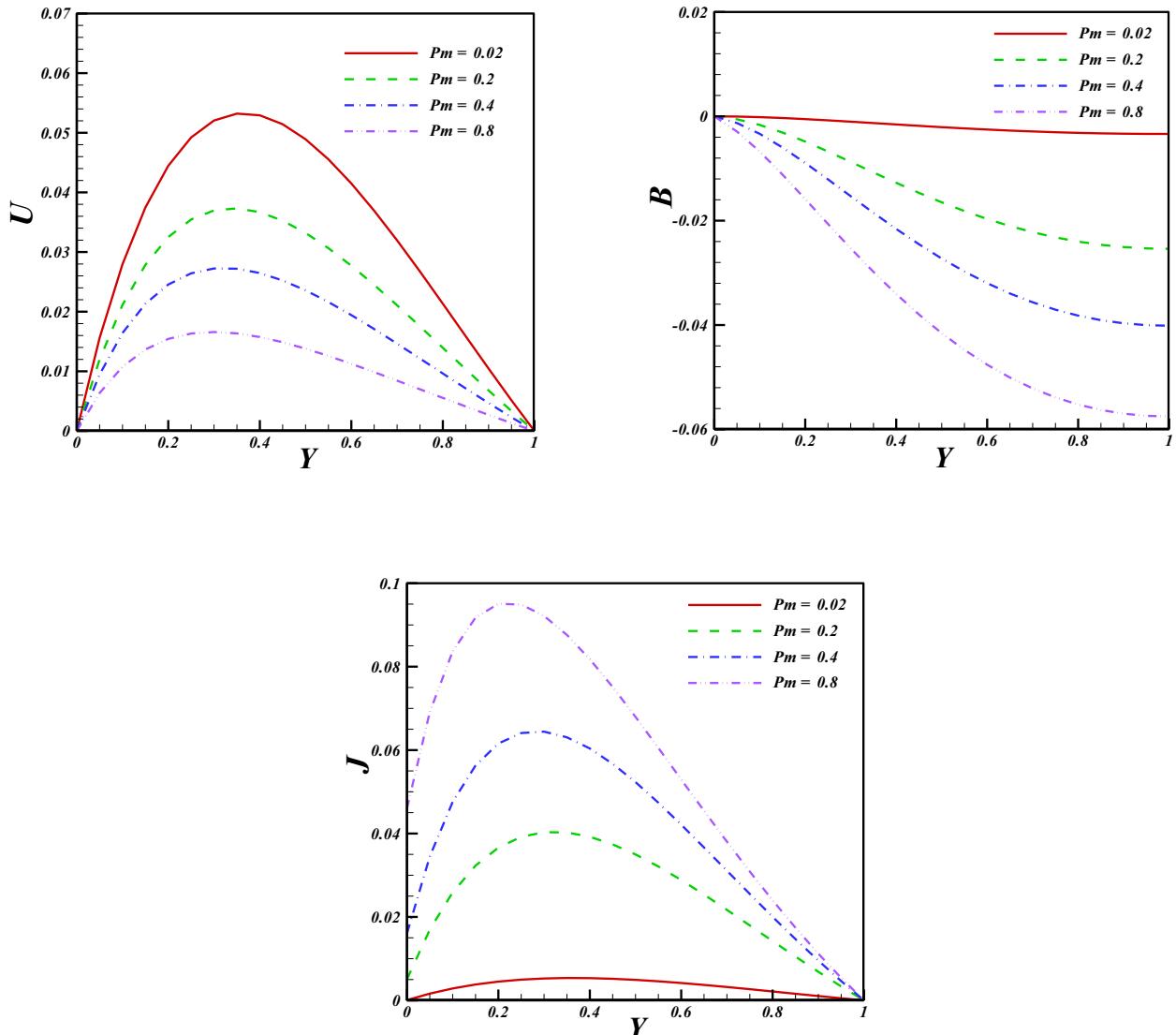


Fig. 4. Effect of magnetic Prandtl number on velocity, induced magnetic field and induced current density distributions when $V_0 = 1$, $Ha = 5$, $Nt = 0.1$, $Nb = 0.1$, $Br = 1$, $Sc = 1$, $Pr = 10$.

$$\begin{pmatrix} 0 \\ 0 \\ u_1 \\ 0 \\ u_2 \\ u_3 \\ -1 \\ 1 \\ u_4 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix} \quad (16)$$

Eqs. (15) and (16) are solved utilizing 4th order Runge-Kutta method. According to $U(1) = 0$, $B'(1) = 0$, $\theta(1) = 0$, $\phi(1) = 0$, unknown initial conditions can be obtained by Newton's method.

4. Results and discussion

Influence of magnetic field on two phase nanofluid double diffusion convective flow between is investigated. MAPLE code has

been validated by comparison with previous published paper [33]. Table 1 illustrates good accuracy of present code. The influences of significant parameters such as Buoyancy ratio, suction parameter, Hartmann and magnetic Prandtl numbers, Schmidt number, Brownian and thermophoretic parameters on mass and heat transfer are presented. Fig. 2 depicts the influence of suction parameter on induced current density, magnetic field, velocity, concentration and temperature distributions. As suction parameter augments velocity, induced magnetic field, induced current density and concentration distributions decrease but temperature profile augments. Fig. 3 depicts the impact of Lorentz force on velocity, induced current density and magnetic field distributions. Nanofluid flow reduces with augment of Ha . Generally, B profile decreases with rise of Lorentz forces but current density augments with augment of this parameter. Fig. 4 depicts the influence of Pm on induced magnetic field, current density and velocity distributions. Pm causes induced magnetic field and velocity to reduce while J augments with rise of Pm . Influences of Buoyancy ratio on velocity, induced current density and magnetic field

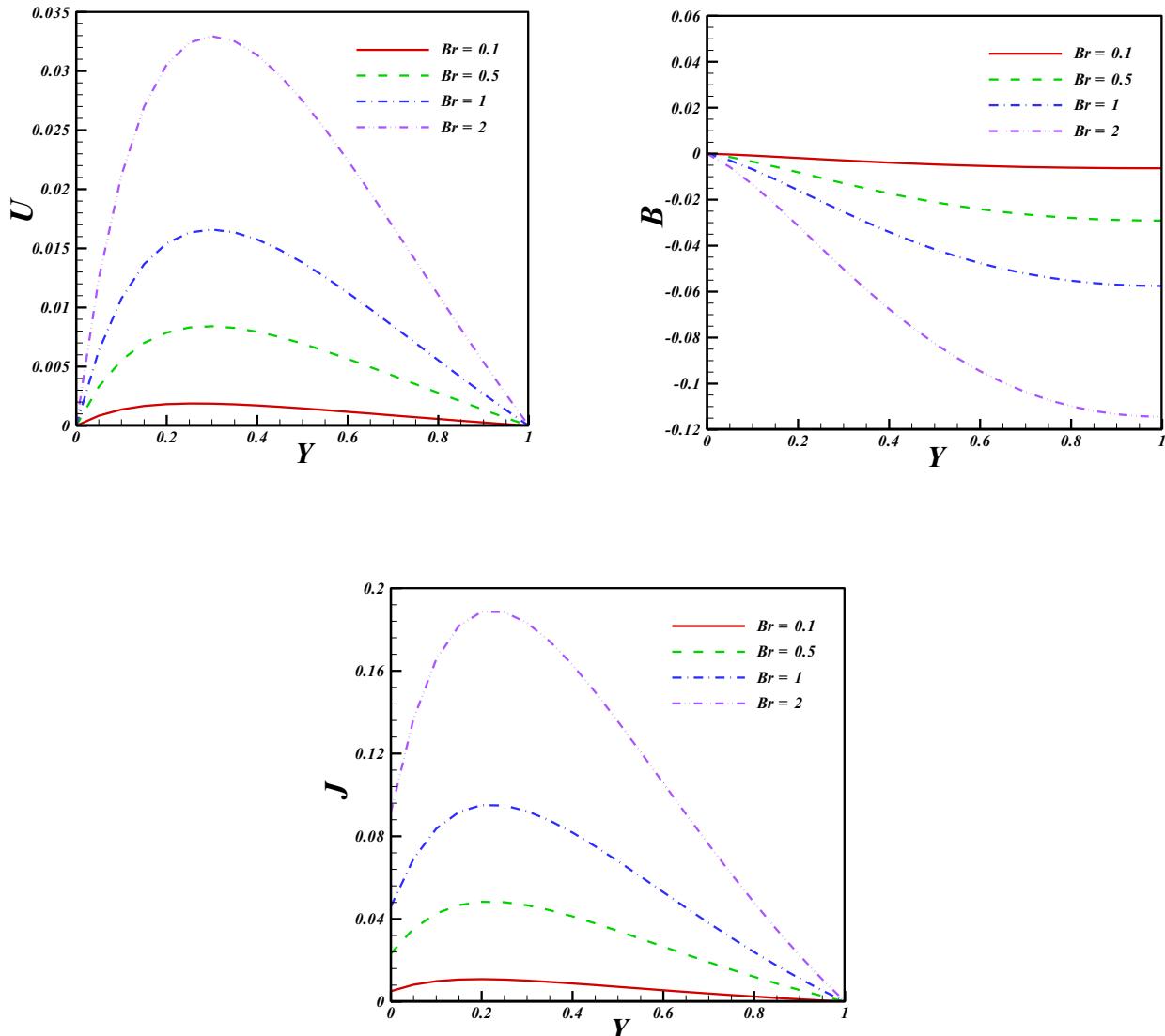


Fig. 5. Effect of Buoyancy ratio on velocity, induced magnetic field and induced current density distributions when $V_0 = 1$, $Ha = 5$, $Pm = 0.8$, $Nt = 0.1$, $Nb = 0.1$, $Sc = 1$, $Pr = 10$.

distributions are illustrated in Fig. 5. Current density and velocity rise with augment of Buoyancy ratio but induced magnetic field decreases with augment of Br .

Figs. 6–8 depict the impact of thermophoretic, Brownian motion parameters and Schmidt number on induced current density and magnetic field, temperature, velocity and concentration distributions. As thermophoretic parameter augments, velocity, current density, temperature and concentration distributions enhance but induced magnetic field and temperature profiles decrease. Effect of Brownian motion parameter is opposite of thermophoretic parameter. Influences of active parameters on C_f and Nu are shown in Figs. 9 and 10 and Tables 2 and 3. Shear stress is a decrease function of Br , V_0 , Ha , Nb , Pm , Sc and increasing function of Nt . Nusselt number enhances with rise of V_0 while it decreases with augment of Nb , Nt , Sc .

5. Conclusion

Magnetohydrodynamic nanofluid double diffusion convection between two vertical permeable plates is studied using two phase model. Impact of induced magnetic field is taken into account. In order to solve governing equations, Runge-Kutta method is applied. The influence of different dimensionless parameters on induced magnetic field, temperature, velocity and concentration distributions are presented. Results indicate that shear stress enhances with rise of thermophoretic parameter but it reduces with augment of other active parameters. Temperature gradient enhances with augment of suction parameter but it reduces with rise of Schmidt number, Brownian motion and thermophoretic parameters.

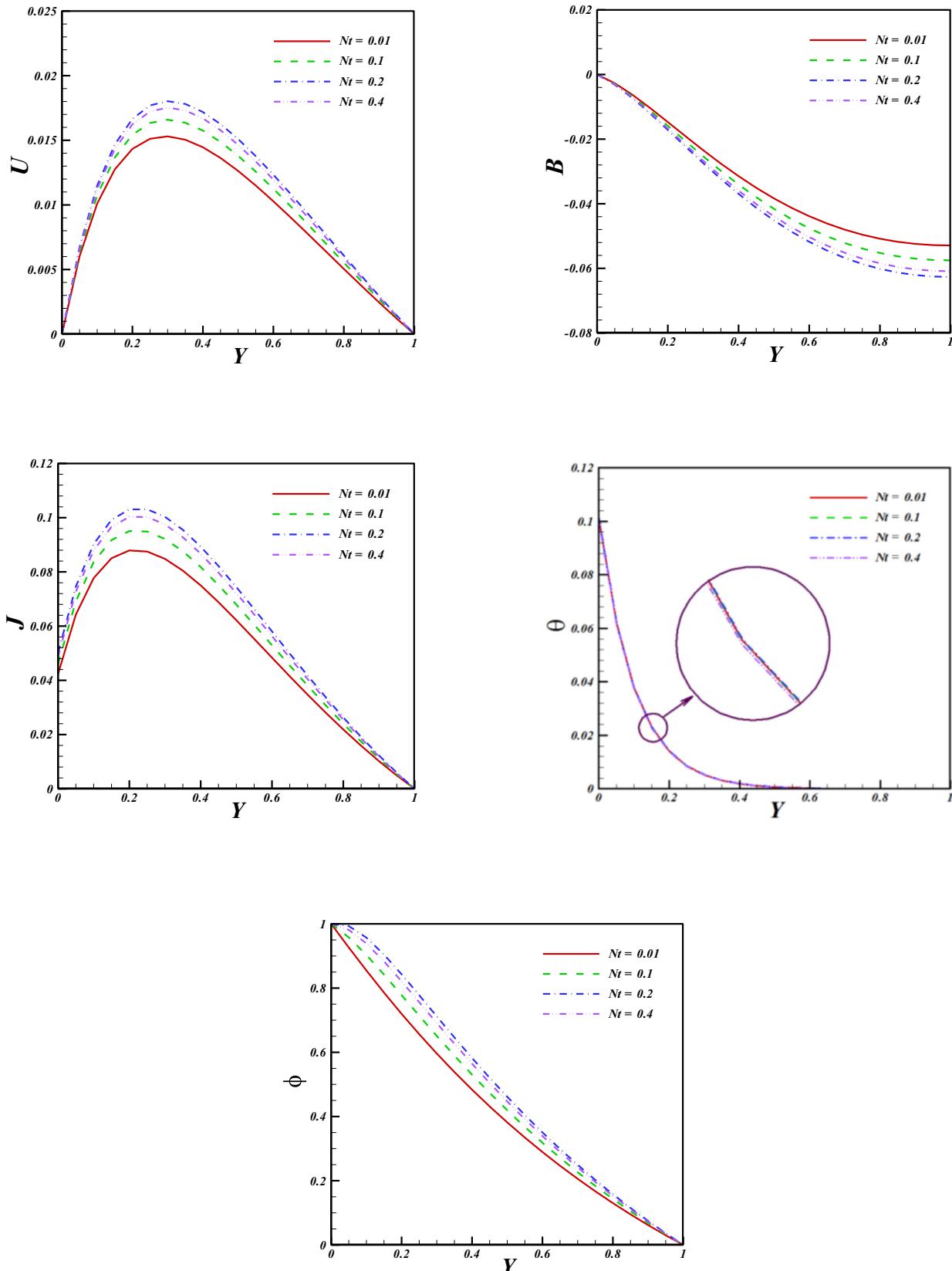


Fig. 6. Effect of thermophoretic parameter on velocity, induced magnetic field, induced current density, temperature and concentration distributions when $V_0 = 1$, $Ha = 5$, $Pm = 0.8$, $Nb = 0.1$, $Sc = 1$, $Pr = 10$.

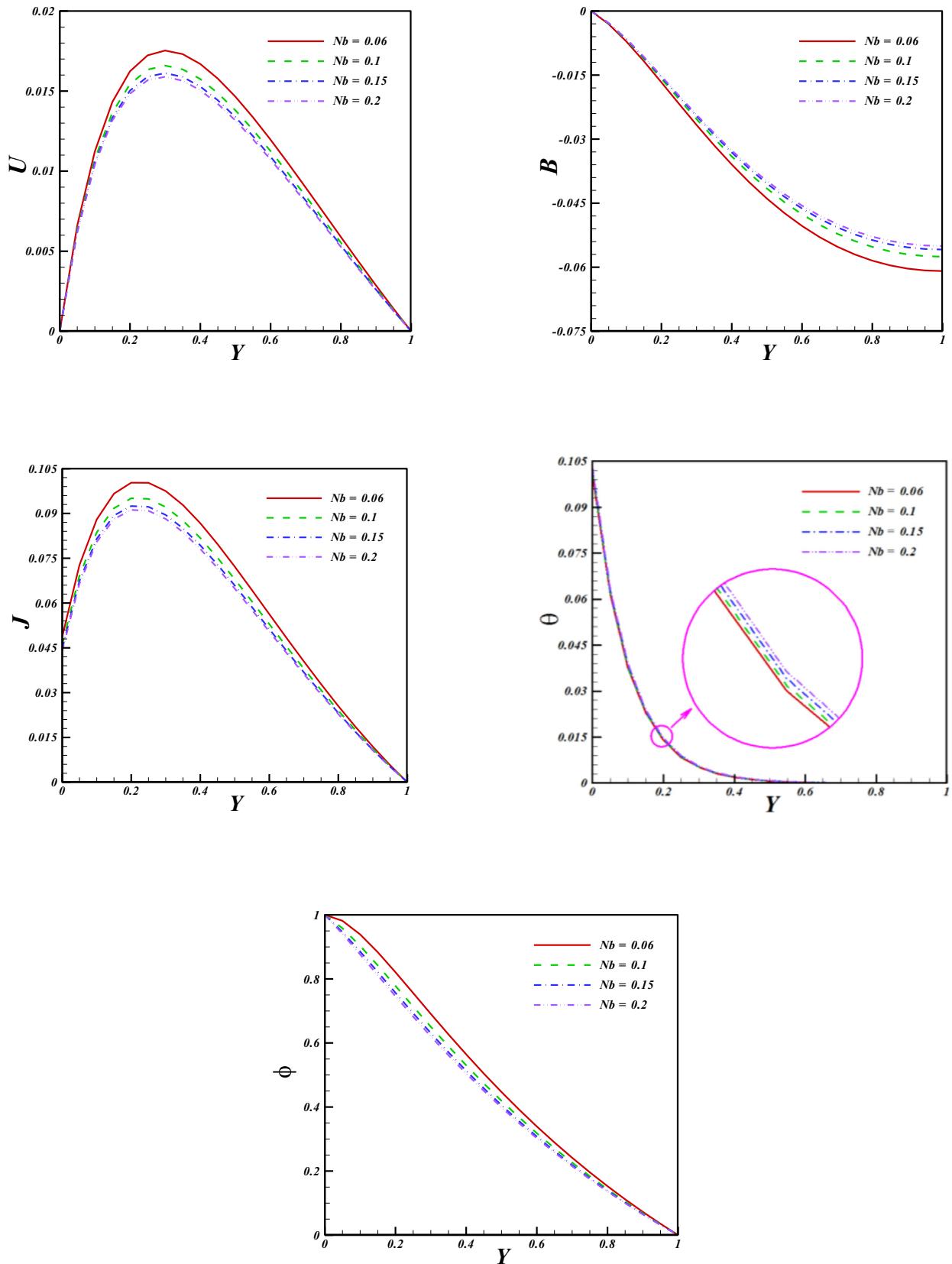


Fig. 7. Effect of Brownian motion parameter on velocity, induced magnetic field, induced current density, temperature and concentration distributions when $V_0 = 1$, $Ha = 5$, $Pm = 0.8$, $Nt = 0.1$, $Sc = 1$, $Pr = 10$.

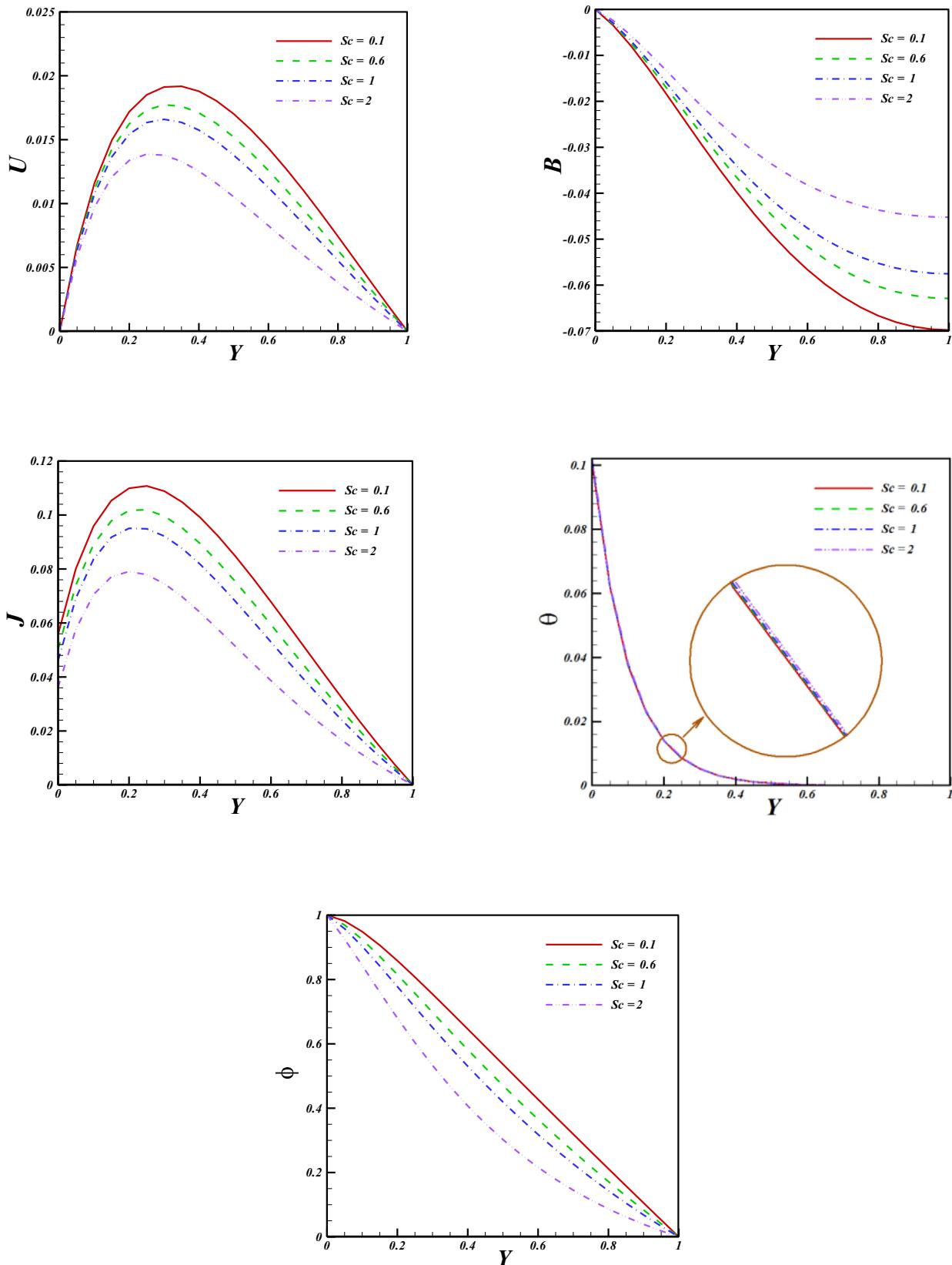


Fig. 8. Effect of Schmidt number on velocity, induced magnetic field, induced current density, temperature and concentration distributions when $V_0 = 1$, $Ha = 5$, $Pm = 0.8$, $Nt = 0.1$, $Nb = 0.1$, $Sc = 1$, $Pr = 10$.

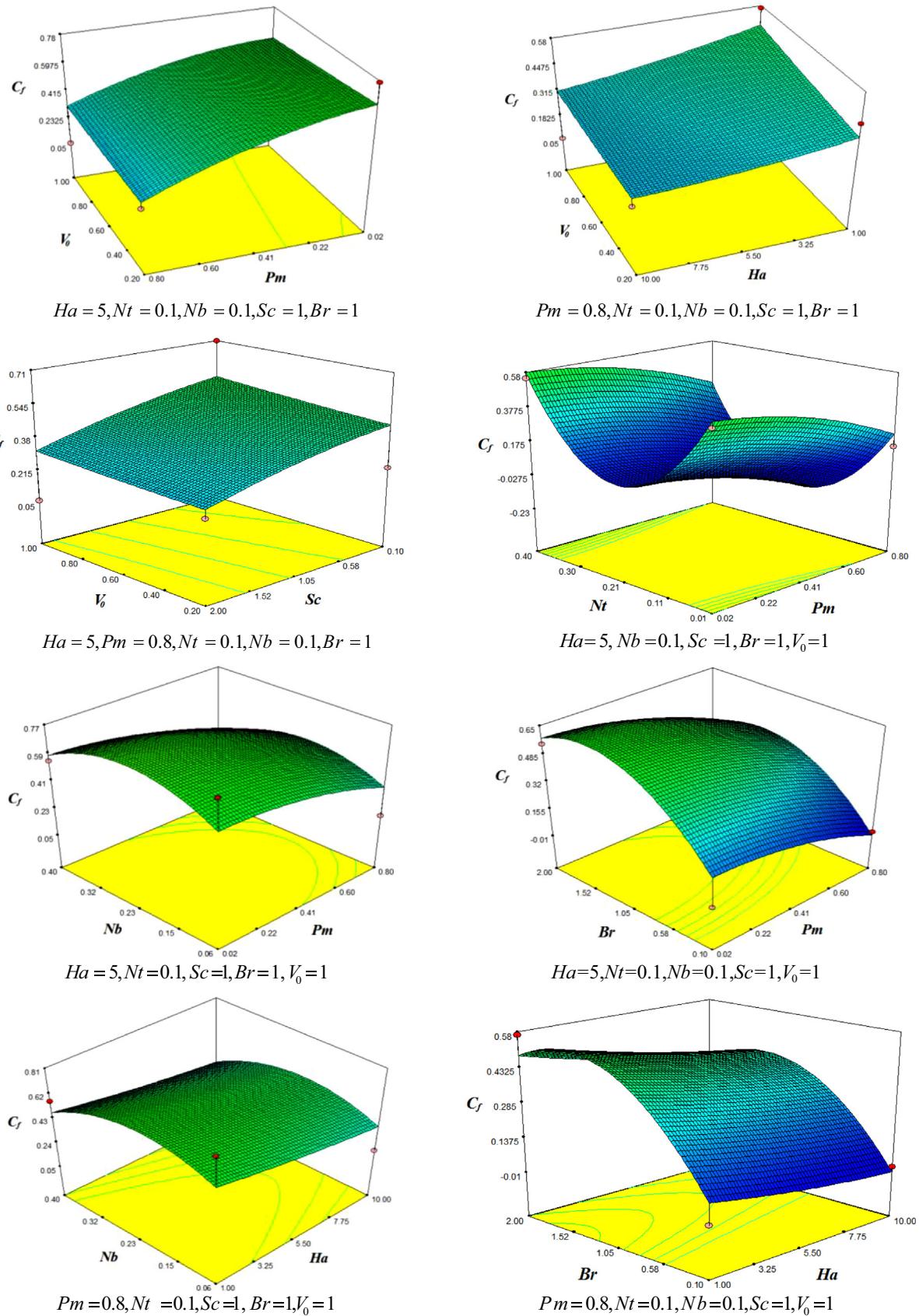
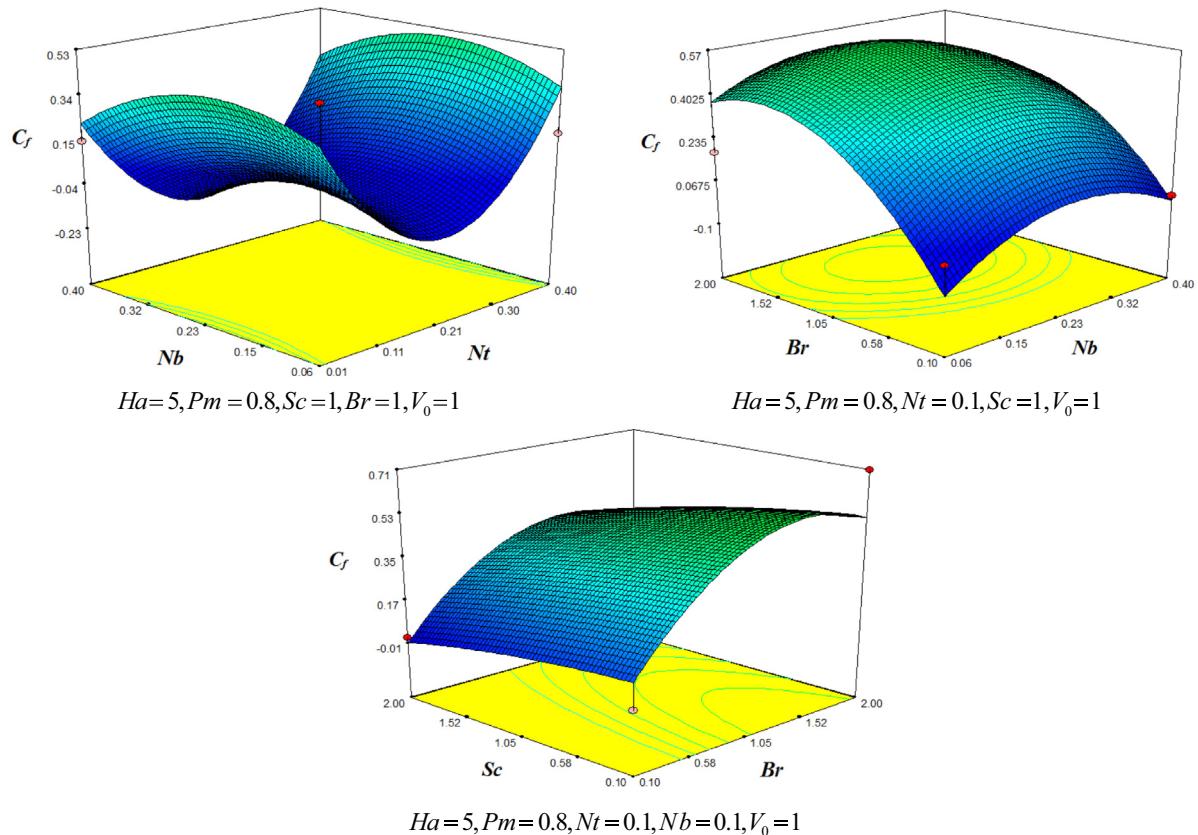
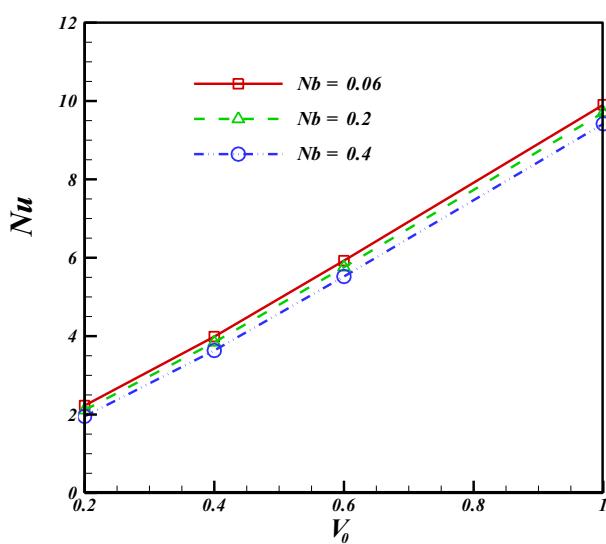


Fig. 9. Effects of active parameters on skin friction coefficient when $\text{Pr} = 10$.

**Fig. 9 (continued)****Fig. 10.** Effects of suction parameter and Brownian motion parameter Nusselt number when $Ha = 5, Pm = 0.8, Nt = 0.1, Br = 1, Sc = 1, \Pr = 10$.**Table 2**

Effects of suction parameter and thermophoretic parameter Nusselt number when $Ha = 5, Pm = 0.8, Nb = 0.1, Br = 1, Sc = 1, \Pr = 10$.

V_0	Nt		
	0.01	0.1	0.4
0.2	2.227115	2.19512	2.082635
0.4	3.967202	3.945781	3.872812
0.6	5.893628	5.878403	5.827099
1	9.855793	9.846286	9.814468

Table 3

Effects of suction parameter and Schmidt number Nusselt number when $Ha = 5, Pm = 0.8, Nt = 0.1, Nb = 0.1, Br = 1, \Pr = 10$.

V_0	Sc		
	0.1	1	2
0.2	2.19832	2.19512	2.191524
0.4	3.955788	3.945781	3.93425
0.6	5.897468	5.878403	5.855804
1	9.88635	9.846286	9.796502

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