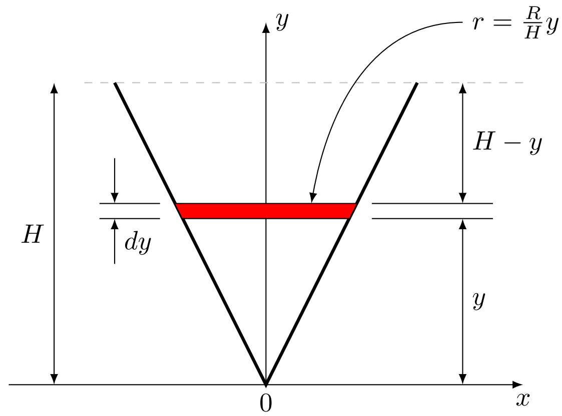

The work (energy) W required to lift the red thin disk to the top of the cone equals (the weight of the disk) $\times$ (the displacement $H - y$). The radius of the disk is $r = \frac{R}{H}y$, so its volume is $\pi\left(\frac{R}{H}y\right)^2 dy$. Let ρ be the water's density. Then the mass of the disk is $\pi\rho\left(\frac{R}{H}y\right)^2 dy$, and so its weight is $\pi\rho g\left(\frac{R}{H}y\right)^2 dy$, where g is the gravitational acceleration. We conclude that the energy required to lift the red disk to the top of the cone is

$$dW = \pi\rho g\left(\frac{R}{H}y\right)^2 (H - y)\, dy.$$

The energy needed to lift the entire contents of the cone is the sum of the energies needed to lift all such disks. The sum is expressed by the integral

$$W = \int_0^H \pi\rho g\left(\frac{R}{H}y\right)^2 (H - y)\, dy.$$

We calculate that integral, either by hand, or by plugging into Maple, and obtain:

$$W = \frac{1}{12}\pi\rho g R^2 H^2.$$

Applying the data $\rho = 1000, R = 1/2, H = 2, g = 9.81$ and obtain $W = 2568.251995$ joules. To match the book's answer, we divide the 1000 to get it as kilo-jouls, and also divide by π to obtain the answer as a multiple of π. We obtain $W = 0.8175\,\pi$ kJ.