$$
\begin{aligned}
sol &= c_1\left(x^4-\frac{1}{12}x^6+\frac{1}{384}x^4+\cdots\right)+c_2\left(\ln x\left(9x^4-\frac{3}{4}x^6+\cdots\right)+\left(-144-36x^2+\frac{1}{2}x^6+\cdots\right)\right)\\
&= c_1\left(x^4-\frac{1}{12}x^6+\frac{1}{384}x^4+\cdots\right)+c_2\left(9\ln x\left(x^4-\frac{1}{12}x^6+\cdots\right)+\left(-144-36x^2+\frac{1}{2}x^6+\cdots\right)\right)
\end{aligned}
$$

Pulling the 9 out and absorbing into c_2 gives

$$
\begin{aligned}
sol &= c_1\left(\overbrace{x^4-\frac{1}{12}x^6+\frac{1}{384}x^4+\cdots}^{y_1}\right)+c_2\left(\ln x\left(\overbrace{x^4-\frac{1}{12}x^6+\cdots}^{y_1}\right)+\left(-\frac{144}{9}-\frac{36}{9}x^2+\frac{1}{18}x^6+\cdots\right)\right)\\
&= c_1\left(x^4-\frac{1}{12}x^6+\frac{1}{384}x^4+\cdots\right)+c_2\left(y_1\ln x+\left(-16-4x^2+\frac{1}{18}x^6+\cdots\right)\right)\\
&= c_1x^4-\frac{c_1}{12}x^6+\frac{c_1}{384}x^4+\cdots+c_2y_1\ln x-16c_2-c_2\left(4x^2\right)+c_2\left(\frac{1}{18}x^6\right)+\cdots\\
&= -16c_2+x^2\left(4c_2\right)+x^4\left(c_1\right)+x^6\left(-\frac{c_1}{12}+c_2\frac{1}{18}\right)+\cdots+c_2y_1\ln x\\
&= -16c_2+x^2\left(4c_2\right)+x^4\left(\frac{1}{2}c_1+\frac{1}{2}c_1\right)+x^6\left(-\frac{c_1}{12}+c_2\frac{1}{18}\right)+\cdots+c_2y_1\ln x\\
&= c_1\left(\frac{1}{2}x^4-\frac{1}{12}x^6+\cdots\right)+c_2\left(-16+\frac{1}{2}x^4+\frac{1}{18}x^6+\cdots\right)+c_2y_1\ln x\\
&= c_1\left(\frac{1}{2}x^4-\frac{1}{12}x^6+\cdots\right)+c_2\left(\ln x\left(x^4-\frac{1}{12}x^6+\cdots\right)+\left(-16+\frac{1}{2}x^4+\frac{1}{18}x^6+\cdots\right)\right)
\end{aligned}
$$

But now the y_1 solution in the front is not correct $c_1\left(\frac{1}{2}x^4-\frac{1}{12}x^6+\cdots\right)$ even though now we managed to get x^4 to show in the y_2 solution, it still does not match the book. May be more playing around is needed.