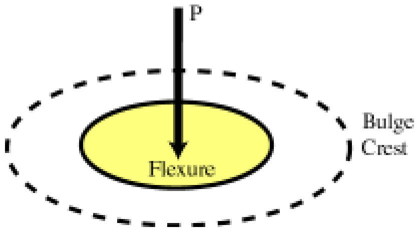


"2.1.BMP"

CALCULATE THE FLEXURE DUE TO AN ARBITRARY-SHAPED LOAD ON A CIRCULAR PLATE USING A FOURIER METHOD

This Mathcad file uses a Fourier method to calculate the flexure of a circular elastic plate caused by a load of arbitrary shape. The load is defined as a grid of values and the flexure is calculated using Mathcad's 2-dimensional Fourier Transform. The flexure can be calculated for any value of load height and width, elastic thickness and load, mantle and infill densities.

Define parameters

$Te := 25km$

$g := 9.81 m \cdot sec^{-2}$

$v := 0.25$

$E := 10^{11} Pa$

$G := 6.67 \cdot 10^{-11} newton \cdot m^2 \cdot kg^{-2}$

$\rho\_load := 2800 kg \cdot m^{-3}$

$\rho\_water := 1030 kg \cdot m^{-3}$

$\rho\_mantle := 3330 kg \cdot m^{-3}$

$\rho\_infill := 2800 kg \cdot m^{-3}$

$npts := 32$

$$D := \frac{E \cdot (Te)^3}{12 \cdot \left(1 - v^2\right)}$$

Create a symmetric diamond-shaped load in the centre of the plate

$load\_height := 5000m$

$load\_width := 50km$

$$grid\_size := \frac{load\_width}{2} \cdot npts$$

$i := 0 .. npts$

$j := 0 .. npts$

$topo_{i,j} := 0.0m$

$$topo_{\left(\frac{npts}{2}, \frac{npts}{2}+1\right)} := load\_height$$

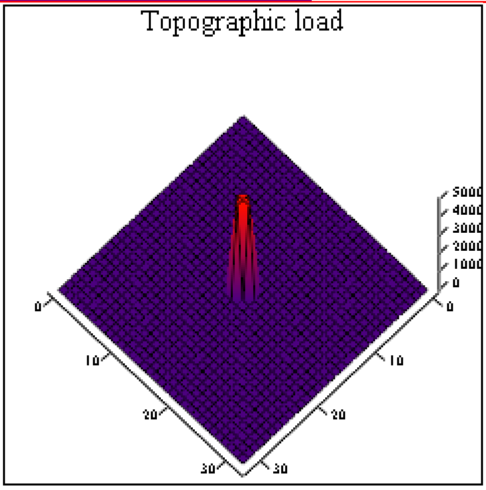
$$topo_{\frac{npts}{2}-1, \frac{npts}{2}} := load\_height$$

$$topo_{\left(\frac{npts}{2}, \frac{npts}{2}\right)} := load\_height$$

$$topo_{\left(\frac{npts}{2}+1, \frac{npts}{2}\right)} := load\_height$$

$$topo_{\left(\frac{npts}{2}, \frac{npts}{2}-1\right)} := load\_height$$

Display the topographic load



topo

$dx := \frac{grid\_size}{npts - 1}$

$dx = 25.806 \cdot km$

$XKINT := \frac{2 \cdot \pi}{npts \cdot dx}$

Take mean out and Fourier transform

$mean\_height := mean(topo)$

$i := 0 .. npts$

$j := 0 .. npts$

$topo_{i,j} := topo_{i,j} - mean\_height$

$c := cfft(topo)$

Calculate elastic plate flexure

$kx := 0, 1 .. \frac{npts}{2}$

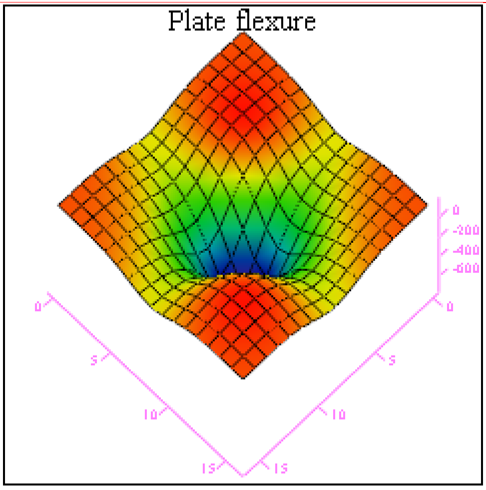
$ky := 0, 1 .. \frac{npts}{2}$

$k_{kx,ky} := \left[(kx \cdot XKINT)^2 + (ky \cdot XKINT)^2\right]^{0.5}$

$Phi_{kx,ky} := \left[\frac{D \cdot \left(k_{kx,ky}\right)^4}{g \cdot (\rho\_mantle - \rho\_infill)} + 1\right]^{-1}$

$flex_{kx,ky} := -(Phi)_{kx,ky} \cdot c_{kx,ky} \cdot \frac{(\rho\_load - \rho\_water)}{(\rho\_mantle - \rho\_infill)}$

$elastic\_flexure := icfft(flex)$



elastic_flexure

grid_size = 800·km

The diagram shows the flexure (negative down) in meters. The "dip" in the bulges between the four quadrants is probably an artifact of the Fourier Transform.