$f := x \rightarrow \sin(x)$

$$x \rightarrow \sin(x) \tag{1}$$

This is a *very simple* periodic function. According to the definition of the Fourier series, the
coefficients can be calculated like this:

$c := n \rightarrow \frac{1}{2\,\pi} \cdot int\left(f(\mathrm{x}) \cdot \mathrm{e}^{-I \cdot n \cdot x}, x = -\pi..\pi\right)$

$$n \rightarrow \frac{1}{2} \frac{\int_{-\pi}^{\pi} f(x)\, \mathrm{e}^{-\mathrm{I}\, n\, x}\, \mathrm{d}x}{\pi} \tag{2}$$

Now, expand f into a Fourier series. g should be equal to f.

$g := x \rightarrow \mathrm{sum}(c(n) \cdot \exp(I \cdot n \cdot x), n = -\infty..\infty);$

$$x \rightarrow \sum_{n=-\infty}^{\infty} c(n)\, \mathrm{e}^{\mathrm{I}\, n\, x} \tag{3}$$

$f\left(\frac{\pi}{2}\right)$

$$1 \tag{4}$$

$g\left(\frac{\pi}{2}\right)$

$$0 \tag{5}$$

Oops. What is this? What's wrong? I am wondering why g(pi/2) = 0 while sin(pi/2) = 1. There is something wrong here.

$c(0)$

$$0 \tag{6}$$

$c(1)$

$$-\frac{1}{2}\,\mathrm{I} \tag{7}$$

$c(-1)$

$$\frac{1}{2}\,\mathrm{I} \tag{8}$$

That's all correct. Both coefficients are correct, too.

$c(-2)$

$$0 \tag{9}$$

$c(2)$

$$0 \tag{10}$$

That's correct, too.

$h := x \rightarrow c(-1) \cdot e^{-I \cdot x} + c(1) \cdot e^{I \cdot x};$

$$x \rightarrow c(-1)\, e^{-I x} + c(1)\, e^{I x} \tag{11}$$

$simplify(h(x));$

$$\sin(x) \tag{12}$$

Yes, that's correct, too.

$evalf\left(h\left(\frac{\pi}{2}\right)\right)$

$$1. \tag{13}$$

Correct, again. It seems that the problem originated in the definition of g using Maple's sum function. Let's try another definition of our Fourier series limiting the summation index range.

$\mathrm{G} := \mathrm{x} \rightarrow \mathrm{sum}(c(n) \cdot \exp(I \cdot n \cdot x), n = -5..5);$

$$x \rightarrow \sum_{n=-5}^{5} c(n)\, e^{I n x} \tag{14}$$

As all coefficients except for n=1 and n=-1 are zero, we can expect G to be equal to f.

$G(0)$

Error, (in SumTools:-DefiniteSum:-ClosedForm) summand is singular in the interval of summation

$G\left(\frac{\pi}{2}\right)$

Error, (in SumTools:-DefiniteSum:-ClosedForm) summand is singular in the interval of summation

$singular(G(x), x)$

Error, (in SumTools:-DefiniteSum:-ClosedForm) summand is singular in the interval of summation

Okay, let us dive into the integral that is used for computing the Fourier series coefficients

$singular\left(\frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} \mathrm{f}(\mathrm{x}) \cdot e^{-I \cdot n \cdot x} \, \mathrm{dx}, \mathrm{n}\right);$

$$\{n = -1\}, \{n = 1\} \tag{15}$$

Aha. So, integration will probably lead to a singularity. Let's look how this integral looks like:

$$\int_{-\pi}^{\pi} \sin(x) \cdot e^{-I \cdot n \cdot x} \, dx$$

$$\frac{\left(e^{2 I \pi n} - 1\right) e^{-I \pi n}}{n^2 - 1} \tag{16}$$

Yikes. There we go. There is a singularity. Yet, we can compute

$$\int_{-\pi}^{\pi} \sin(x) \cdot e^{-I \cdot x} \, dx$$

$$-I \pi \tag{17}$$

$$\int_{-\pi}^{\pi} \sin(x) \cdot e^{I \cdot x} \, dx$$

$$I \pi \tag{18}$$

No problem at all, Maple seems to figure out that it just has to take the limit in case of n=1,n=-1. But when using these functions in a sum, Maple will complain and probably not automatically compute the limit. Giving an error is okay. But just producing g(pi/2) = 0 which is a *wrong answer* is not acceptable. Given that my maths is correct, this behaviour should be considered a bug.

Note that adding the assumption 'assume(n::integer)' at the beginning of this Matlab document changes behaviour. Instead of giving errors concerning singularities in a summand term, Maple will just calculate G(pi/2) = 0 without hesitation. This is not correct.

This documented has been tested with Maple 13.0, April 13 2009. I am not a very experienced Maple user. This issue might be related to the Q&A on http://www.mapleprimes.com/questions/37576-summand-Is-Singular