Comparing Mathematica 9 with Maple 17 in solving the 1390 ODEs of Kamke's book

Kamke ODEs linear: 563 nonlinear: 827	Mathematica 9				Maple 17			
	Hangs	Failed to solve	Solved	Time	Hangs	Failed to solve	Solved	Time
1st order: 576 solvable: 547	65	74	86%	250 min	2	6	99%	19 min
2nd order nonlinear: 188 solvable 162	8	48	70%	63 min	2	5	97%	13 min
2nd order nonlinear reducible 58	1	58	0%	23 min	0	8	86%	3 min
2nd order linear 448 solvable: 427	8	40	91%	40 min	1	3	99%	4 min
3rd order linear: 85 solvable: 80	4	22	72%	28 min	1	7	91%	2.5 min
4th order linear: 45 solvable: 43	1	10	77%	8 min	0	3	93%	22 sec
5th order linear: 12	2	3	75%	13 min	0	1	92%	24 sec
High order nonlinear: 12 solvable: 11	1	5	54%	2.5 min	0	0	100%	36 sec
High order nonlinear reducible: 5	0	5	0%	30 sec	0	0	100%	10.5 sec
Totals for the 1390 equations from which 1345 are solvable	90	256	80%	7.1 hours	6	33	97.5%	43 min
Performance in solving Kamke's ODEs Mathematica 9 solved 80% Maple 17 solved 97.5%								

Summary:

- *Mathematica 9 took 7 hours and 8 minutes to tackle the 1390 ODEs, solved 80% of the solvable ODEs, failed with other 256 solvable ODEs and hanged with 90 ODEs.*
- *Maple 17 took 43 minutes to tackle the 1390 ODEs, solved 97.5% of the solvable ODEs, failed with other 31 solvable ODEs and hanged with 6 ODEs.*

Details

The 1390 Ordinary Differential Equations (ODE) of Kamke's book were tackled with Mathematica 9.0.1 DSolve and Maple 17.02 dsolve, with a timelimit of 3 minutes per ODE (Maple function timelimit and Mathematica function TimeConstrained), in a Macbook Pro (2013) 2.7 GHz Intel Core I7, 4 cores, 16 GB of RAM. No optional arguments were allowed when calling DSolve and dsolve.

The test focused in: time consumed, number of equations solved versus unsolved, and also number of "hangs" as a measure of the ability of the system to understand that a problem is beyond its capabilities.

In this report we say that the ODE solver hanged with an equation when it did not terminate before the three minutes time limit and so the computation got automatically interrupted, with the system moving to tackling the next equation.

By "unsolved" equations it is meant equations for which we know a solution exists (either because Kamke shows it or because we were able to derive it and test it), and for which the ODE solver terminates before 3 minutes, and fails in finding any solution, or it hanged. Equations for which the answer returned involved DESol in Maple and the equivalent DifferentialRoot in Mathematica are considered unsolved in that the answer returned does not involve more information than the equation passed to be solved.

Regarding the unsolvable equations of Kamke's book, naturally, neither Mathematica nor Maple are expected to solve them, but both systems are expected not to hang with them.

To compare the time spent in tackling the 1390 ODEs discarding the hangs, multiply the number of hangs by 3 min and subtract that from the total time consumed.

First order ODEs

There are 576 first order ODEs in Kamke's book: 29 are unsolvable and 547 are solvable. The Kamke's numbers for the unsolvable ODEs are 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 367, 370, 395, 461, 503, 572, 576.

Mathematica 9 performance: 86.5 %, in 4 hours and 10 minutes

Mathematica took 4 hours and 10 minutes in tackling the 576 ODEs, solved 473 of the 547 solvable ones and failed in returning a solution for 74 solvable equations: 16, 22, 63, 66, 69, 70, 80, 81, 83, 86, 121, 127, 266, 292, 340, 365, 366, 368, 383, 385, 394, 400, 402, 404, 413, 414, 416, 428, 429, 451, 452, 460, 465, 467, 468, 479, 482, 485, 487, 489, 494, 504, 506, 508, 509, 510, 513, 515, 520, 523, 524, 527, 528, 530, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 555, 561, 562, 566, 567, 570, 575. Independent of that, Mathematica hangs with 65 equations: 22, 63, 66, 69, 70, 86, 127, 219, 250, 265, 266, 269, 292, 340, 365, 366, 368, 383, 385, 400, 402, 404, 413, 414, 416, 428, 429, 451, 452, 465, 467, 468, 479, 482, 485, 487, 489, 494, 503, 504, 509, 513, 515, 520, 523, 524, 527, 528, 530, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 555, 561, 562, from which 60 equations are solvable.

Maple 17 performance: 99 %, in 19 minutes

Maple took 19 minutes in tackling the 576 ODEs, solved 541 of the 547 solvable ones and failed in returning a solution for 6 solvable equations: 121, 340, 460, 506, 510, 575. Independent of that, Maple hangs with 2 equations: 331, 340, from which 340 is solvable.

Second order nonlinear ODEs

There are 246 second order nonlinear ODEs in Kamke's book, from which 26 are unsolvable and 220 are solvable, divided into fully solvable (162 of them) and reducible (58 of them).

The Kamke's numbers of the 26 unsolvable ODEs are: 3, 5, 6, 8, 9, 13, 16, 18, 19, 27, 29, 52, 55, 59, 85, 95, 112, 114, 139, 147, 149, 161, 207, 211, 212, 217.

The Kamke numbers of the 58 second order nonlinear ODEs that are only reducible are: 11, 15, 21, 22, 25, 26, 28, 34, 36, 37, 44, 46, 47, 49, 53, 54, 58, 68, 69, 70, 72, 73, 74, 75, 76, 77, 82, 83, 87, 90, 92, 94, 96, 100, 102, 103, 105, 106, 115, 116, 118, 123, 129, 131, 144, 148, 152, 172, 187, 198, 199, 225, 226, 230, 231, 235, 242, 246.

Maple conveys solutions for the reducible equations using 'ODESolStruc', showing the solution for the original problem in terms of the solution of the reduced ODE, as well as the reduced ODE and transformation used to reduce the problem. Mathematica does not convey reductions of order, thus failing with all the 58 equations of 'only reducible to first order' type.

Mathematica 9 performance: solvable: 70.4 %, reducible 0 %, in 1 hour and 16 minutes

Mathematica took 1 hour and 16 minutes in tackling the 246 ODEs, solved 114 of the 162 solvable ones, 0 of the 58 reducible ones, and failed in returning a solution for 48 fully solvable equations: 23, 24, 30, 31, 32, 33, 35, 38, 39, 41, 43, 45, 48, 62, 66, 88, 91, 101, 108, 119, 120, 130, 142, 145, 156, 165, 167, 171, 189, 190, 208, 216, 219, 223, 227, 228, 229, 233, 237, 241, 244, 20, 50, 98, 121, 186, 221, 243. Independent of that, Mathematica hangs with 8 equations: 20, 50, 98, 121, 186, 221, 230, 243, from which 7 equations are fully solvable.

Maple 17 performance: solvable: 97 %, reducible: 86 %, in 16 minutes

Maple took 16 minutes in tackling the 246 ODEs, solved 157 of the 162 solvable ones, 50 of the 58 reducible ones, and failed in returning a solution for 5 fully solvable equations: 108, 142, 145, 167, 171. Independent of that, Maple hangs with 2 equations: 217, 245, from which 1 equation is fully solvable.

Second order linear ODEs

There are 448 second order linear ODEs in Kamke's book, from which 21 are unsolvable and 427 are solvable. The Kamke's numbers for the unsolvable ODEs are 15, 19, 26, 28, 30, 31, 38, 72, 73, 75, 76, 77, 205, 212, 216, 236, 278, 408, 439, 440, 443.

Mathematica 9 performance: 91 %, in 40 minutes

Mathematica took 40 minutes in tackling the 448 ODEs, solved 387 of the 427 solvable ones and failed in returning a solution for 40 solvable equations: 27, 29, 32, 80, 81, 82, 83, 84, 85, 128, 156, 413, 418, 442, 445, 157, 248, 261, 268, 303, 306, 329, 330, 343, 348, 362, 367, 372,

373, 398, 402, 403, 177, 232, 233, 263, 341, 406, 407, 427. Independent of that, Mathematica hangs with 8 equations: 177, 232, 233, 263, 341, 406, 407, 427, all of which are solvable .

Maple 17 performance: 99 %, in 4 minutes

Maple took 4 minutes in tackling the 448 ODEs, solved 424 of the 427 solvable ones and failed in returning a solution for 3 solvable equations: 81, 57, 441. Maple hangs with 1 equation: 441.

Third order linear ODEs

There are 85 third order linear ODEs in Kamke's book, from which 5 are unsolvable and 80 are solvable. The Kamke's numbers for the unsolvable ODEs are 9, 10, 11, 12, 28.

Mathematica 9 performance: 72.5 %, in 28 minutes

Mathematica took 28 minutes in tackling the 85 ODEs, solved 58 of the 80 solvable ones and failed in returning a solution for 22 solvable equations: 13, 14, 15, 23, 24, 25, 26, 59, 62, 67, 81, 82, 83, 36, 41, 57, 72, 79, 2, 34, 68, 78. Independent of that, Mathematica hangs with 4 equations: 2, 34, 68, 78, all of which are solvable.

Maple 17 performance: 91 %, in 2.5 minutes

Maple took 2.5 minutes in tackling the 85 ODEs, solved 73 of the 80 solvable ones and failed in returning a solution for 7 solvable equations: 13, 14, 25, 36, 41, 62, 83. Independent of that, Maple hangs with 1 equation: 67, which is solvable.

Fourth order linear ODEs

There are 45 fourth order linear ODEs in Kamke's book, from which 2 are unsolvable and 43 are solvable. The Kamke's numbers for the unsolvable ODEs are 7, 8.

Mathematica 9 performance: 76.7 %. in 8 minutes

Mathematica took 8 minutes in tackling the 45 ODEs, solved 33 of the 43 solvable ones and failed in returning a solution for 10 solvable equations: 9, 10, 11, 14, 40, 41, 42, 43, 39, 19. Independent of that, Mathematica hangs with 1 equation: 19, which is solvable.

Maple 17 performance: 93 %, in 22 seconds

Maple took 22 seconds in tackling the 45 ODEs, solved 40 of the 43 solvable ones and failed in returning a solution for 3 solvable equations: 9, 10, 11. Independent of that, Maple did not hang with any equation.

Fifth order linear ODEs

There are 12 fifth order linear ODEs in Kamke's book, all of them solvable.

Mathematica 9 performance: 75 %, in 13 minutes

Mathematica took 13 minutes in tackling the 12 ODEs, solved 9 of the 12 solvable equations and failed in returning a solution for 3 solvable equations: 8, 5, 12. Independent of that, Mathematica hangs with 2 equation: 5, 12, which are both solvable.

Maple 17 performance: 92 %, in 24 seconds

Maple took 24 seconds in tackling the 12 ODEs, solved 11 of the 12 solvable equations and failed in returning a solution for 1 solvable equation: 4. Independent of that, Maple did not hang with any equation.

Higher order nonlinear ODEs

There are 17 higher order nonlinear ODEs in Kamke's book, from which 1 is unsolvable and 16 are solvable, in turn divided into fully solvable (11 of them) and fully reducible (5 of them). The Kamke's number for the unsolvable ODE is 15.

The Kamke numbers of the 5 higher order ODEs that are only reducible are: 2, 3, 4, 5, 14.

Mathematica 9 performance with solvable ODEs: 54.6 %; with reducible ODEs: 0 %, in 3 minutes

Mathematica took 3 minutes in tackling the 17 ODEs, solved 6 of the 11 solvable ones, 0 of the 5 reducible ones, and failed in returning a solution for 5 fully solvable equations: 1, 8, 9, 17, 12. Independent of that, Mathematica hangs with 1 equation: 12, which is fully solvable.

Maple 17 performance with solvable ODEs: 100%; with reducible ODEs: 100%, in 24 seconds

Maple took 24 seconds in tackling the 17 ODEs, solved 11 of the 11 solvable ones, 5 of the 5 reducible ones, so all of them. Independent of that, Maple did not hang with any equation.