

with (PDEtools, casesplit, declare)

[casesplit, declare]

with (DEtools, gensys)

[gensys]

$[\xi_1(x, t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t), a(t), b(t), c(t)]$

alias ($\xi_1 = \xi_1(x, t), \xi_2 = \xi_2(y, z), \xi_3 = \xi_3(y, z), \xi_4 = \xi_4(t), \eta = \eta(x, y, z, t, u), a = a(t), b = b(t), c = c(t)$)

$\xi_1, \xi_2, \xi_3, \xi_4, \eta, a, b, c$

declare ($\xi_1(x, t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u), a(t), b(t), c(t)$)

$\eta(x, y, z, t, u)$ 'will now be displayed as' η

$a(t)$ 'will now be displayed as' a

$b(t)$ 'will now be displayed as' b

$c(t)$ 'will now be displayed as' c

$\xi(t)$ 'will now be displayed as' ξ

$\xi(x, t)$ 'will now be displayed as' ξ

$\xi(y, z)$ 'will now be displayed as' ξ

sys := $[\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dxdu}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2c\frac{d}{dz}\xi_3 - 2\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc -$

$\frac{d^2}{dy^2}\xi_2(y, z) = 0, 2c(t)\frac{\partial^2}{\partial u\partial z}\eta(x, y, z, t, u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y, z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y, z) = 0, a(t)\left(\frac{\partial}{\partial x}\eta(x, y, z, t, u)\right)u + b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x, y, z, t, u) -$

$$\frac{\partial^2}{\partial x^2}\xi_1(x, t) = 0$$

$$\frac{\partial^2}{\partial u^2}\eta(x, y, z, t, u) = 0$$

$$\frac{\partial^2}{\partial u\partial x}\eta(x, y, z, t, u) = 0$$

$$c(t)\frac{\partial}{\partial z}\xi_2(y, z) + b(t)\frac{\partial}{\partial y}\xi_3(y, z) = 0$$

$$-\xi_4\frac{d}{dt}c(t) + 2c(t)\frac{\partial}{\partial z}\xi_3(y, z) - 2\frac{\partial}{\partial x}\xi_1(x, t) = 0$$

$$2\left(\frac{\partial}{\partial z}\xi_3(y, z)\right)b(t)c(t) - 2\left(\frac{\partial}{\partial y}\xi_2(y, z)\right)b(t)c(t) - b(t)\xi_4\frac{d}{dt}c(t) + \xi_4c(t)\frac{d}{dt}b(t) = 0$$

$$-b(t)\frac{\partial^2}{\partial y^2}\xi_2(y, z) + 2b(t)\frac{\partial^2}{\partial u\partial y}\eta(x, y, z, t, u) - c(t)\frac{\partial^2}{\partial z^2}\xi_2(y, z) = 0$$

$$2c(t)\frac{\partial^2}{\partial u\partial z}\eta(x, y, z, t, u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y, z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y, z) = 0$$

$$\begin{aligned}
& a(t) \left(\frac{\partial}{\partial x} \eta(x, y, z, t, u) \right) u + b(t) \frac{\partial^3}{\partial x \partial y^2} \eta(x, y, z, t, u) + c(t) \frac{\partial^3}{\partial x \partial z^2} \eta(x, y, z, t, u) + \frac{\partial}{\partial t} \eta(x, y, z, t, u) + \frac{\partial^3}{\partial x^3} \eta(x, y, z, t, u) = 0 \\
& -\xi_4 \frac{d}{dt} c(t) - c(t) \frac{d}{dt} \xi_4(t) + 2c(t) \frac{\partial}{\partial z} \xi_3(y, z) + c(t) \frac{\partial}{\partial x} \xi_1(x, t) = 0 \\
& \eta(x, y, z, t, u) + (c(t))^2 \frac{\partial^3}{\partial u \partial z^2} \eta(x, y, z, t, u) - c(t) \frac{\partial}{\partial t} \xi_1(x, t) + 2a(t)c(t) \left(\frac{\partial}{\partial z} \xi_3(y, z) \right) u - u \left(a(t) \frac{d}{dt} c(t) - c(t) \frac{d}{dt} a(t) \right) \xi_4 + \eta(x, y, z, t, u) \\
& \text{nops} \left(\left[\frac{d^2}{dx^2} \xi_1 = 0, \frac{d^2}{du^2} \eta = 0, \frac{d^2}{dx du} \eta = 0, c \frac{d}{dz} \xi_2 + b \frac{d}{dy} \xi_3 = 0, -\xi_4 \frac{d}{dt} c + 2c \frac{d}{dz} \xi_3 - 2 \frac{d}{dx} \xi_1 = 0, 2 \left(\frac{d}{dz} \xi_3 \right) bc - \right. \right. \\
& \left. \left. \frac{\partial^2}{\partial z^2} \xi_2(y, z) = 0, 2c \frac{\partial^2}{\partial u \partial z} \eta(x, y, z, t, u) - b \frac{\partial^2}{\partial y^2} \xi_3(y, z) - c \frac{\partial^2}{\partial z^2} \xi_3(y, z) = 0, a \left(\frac{\partial}{\partial x} \eta(x, y, z, t, u) \right) u + b \frac{\partial^3}{\partial x \partial y^2} \eta(x, y, z, t, u) + c \frac{\partial^3}{\partial x^3} \eta(x, y, z, t, u) \right] \right)
\end{aligned}$$

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restart

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with (DEtools, gensys)

[gensys]

[$\xi_1(t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u)$]

alias ($\xi_1 = \xi_1(t), \xi_2 = \xi_2(y, z), \xi_3 = \xi_3(y, z), \xi_4 = \xi_4(t), \eta = \eta(x, y, z, t, u)$)

$\xi_1, \xi_2, \xi_3, \xi_4, \eta$

declare ($\xi_1(t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u)$)

$\eta(x, y, z, t, u)$ 'will now be displayed as' η

$\xi(t)$ 'will now be displayed as' ξ

$\xi(y, z)$ 'will now be displayed as' ξ

sys1 := [$\frac{d^2}{dx^2} \xi_1 = 0, \frac{d^2}{du^2} \eta = 0, \frac{d^2}{dx du} \eta = 0, c \frac{d}{dz} \xi_2 + b \frac{d}{dy} \xi_3 = 0, -\xi_4 \frac{d}{dt} c + 2c \frac{d}{dz} \xi_3 - 2 \frac{d}{dx} \xi_1 = 0, 2 \left(\frac{d}{dz} \xi_3 \right) bc -$

$\frac{\partial^2}{\partial z^2} \xi_2(y, z) = 0, 2c \frac{\partial^2}{\partial u \partial z} \eta(x, y, z, t, u) - b \frac{\partial^2}{\partial y^2} \xi_3(y, z) - c \frac{\partial^2}{\partial z^2} \xi_3(y, z) = 0, a \left(\frac{\partial}{\partial x} \eta(x, y, z, t, u) \right) u + b \frac{\partial^3}{\partial x \partial y^2} \eta(x, y, z, t, u) + c \frac{\partial^3}{\partial x^3} \eta(x, y, z, t, u)$

sol := *pdsolve* (*sys1*, [$\xi_1, \xi_2, \xi_3, \xi_4, \eta$])

sol := $\left\{ \xi_1 = -C1 at + -C6, \xi_4 = -C2, \eta(x, y, z, t, u) = -C1, \xi_2(y, z) = -\frac{-C3 bz}{c} + -C5, \xi_3(y, z) = -C3 y + -C4 \right\}$

restart

with (PDEtools, casesplit, declare)

[casesplit, declare]

with (DEtools, gensys)

[gensys]

[$\xi_1(t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u), a(t), b(t), c(t)$]

alias ($\xi_1 = \xi_1(t), \xi_2 = \xi_2(y, z), \xi_3 = \xi_3(y, z), \xi_4 = \xi_4(t), \eta = \eta(x, y, z, t, u), a = a(t), b = b(t), c = c(t)$)

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        ξ1, ξ2, ξ3, ξ4, η, a, b, c
declare (ξ1(t), ξ2(y, z), ξ3(y, z), ξ4(t), η(x, y, z, t, u), a(t), b(t), c(t))
        η(x, y, z, t, u) 'will now be displayed as' η
        a(t) 'will now be displayed as' a
        b(t) 'will now be displayed as' b
        c(t) 'will now be displayed as' c
        ξ(t) 'will now be displayed as' ξ
        ξ(y, z) 'will now be displayed as' ξ
sys2 := [d2/dx2 ξ1 = 0, d2/du2 η = 0, d2/dxdu η = 0, c d/dz ξ2 + b d/dy ξ3 = 0, -ξ4 d/dt c + 2 c d/dz ξ3 - 2 d/dx ξ1 = 0, 2 (d/dz ξ3) b c
ξ2(y, z) = 0, 2 c(t) d2/du dz η(x, y, z, t, u) - b(t) d2/dy2 ξ3(y, z) - c(t) d2/dz2 ξ3(y, z) = 0, a(t) (∂/∂x η(x, y, z, t, u)) u + b(t) d3/dx dy2 η(x, y, z, t, u) +
sol := pdsolve(sys2, [ξ1, ξ2, ξ3, ξ4, η, a, b, c])
-F5(y, z), ξ2(y, z) = ξ2(y, z), ξ3(y, z) = ξ3(y, z) } , [ { ξ1 = ξ1, ξ4 = ξ4, a(t) = 0, c(t) = 0, ξ2(y, z) = ∫ 2 d/dx η(x, y, z, t, u) dy + -F2(z) y
restart

with (PDEtools, casesplit, declare)
        [casesplit, declare]
with (DEtools, gensys)
        [gensys]

[ξ1(x, t), ξ2(y, z), ξ3(y, z), ξ4(t), η(x, y, z, t, u)]
alias (ξ1 = ξ1(x, t), ξ2 = ξ2(y, z), ξ3 = ξ3(y, z), ξ4 = ξ4(t), η = η(x, y, z, t, u))
        ξ1, ξ2, ξ3, ξ4, η
declare (ξ1(x, t), ξ2(y, z), ξ3(y, z), ξ4(t), η(x, y, z, t, u))
        η(x, y, z, t, u) 'will now be displayed as' η
        ξ(t) 'will now be displayed as' ξ
        ξ(x, t) 'will now be displayed as' ξ
        ξ(y, z) 'will now be displayed as' ξ
sys3 := [d2/dx2 ξ1 = 0, d2/du2 η = 0, d2/dxdu η = 0, c d/dz ξ2 + b d/dy ξ3 = 0, -ξ4 d/dt c + 2 c d/dz ξ3 - 2 d/dx ξ1 = 0, 2 (d/dz ξ3) b c
ξ2(y, z) = 0, c d2/dz2 ξ2(y, z) = 0, 2 c d2/du dz η(x, y, z, t, u) - b d2/dy2 ξ3(y, z) - c d2/dz2 ξ3(y, z) = 0, a (∂/∂x η(x, y, z, t, u)) u + b d3/dx dy2 η(x, y, z, t, u) +
sol := pdsolve(sys3, [ξ1, ξ2, ξ3, ξ4, η])
t - 1/2 -C1 xc + -C7, ξ4 = -1/2 -C1 (c + 2) t + -C3, η(x, y, z, t, u) = -C1 u + -C2, ξ2(y, z) = -1/2 -C1 y - b -C4 z / c + -C6, ξ3(y, z) = -1/2

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restart

with (PDEtools, casesplit, declare)

[*casesplit, declare*]

with (DEtools, gensys)

[*gensys*]

[$\xi_1(x, t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u), a(t), b(t), c(t)$]

alias ($\xi_1 = \xi_1(x, t), \xi_2 = \xi_2(y, z), \xi_3 = \xi_3(y, z), \xi_4 = \xi_4(t), \eta = \eta(x, y, z, t, u), a = a(t), b = b(t), c = c(t)$)

$\xi_1, \xi_2, \xi_3, \xi_4, \eta, a, b, c$

declare ($\xi_1(x, t), \xi_2(y, z), \xi_3(y, z), \xi_4(t), \eta(x, y, z, t, u), a(t), b(t), c(t)$)

$\eta(x, y, z, t, u)$ 'will now be displayed as' η

$a(t)$ 'will now be displayed as' a

$b(t)$ 'will now be displayed as' b

$c(t)$ 'will now be displayed as' c

$\xi(t)$ 'will now be displayed as' ξ

$\xi(x, t)$ 'will now be displayed as' ξ

$\xi(y, z)$ 'will now be displayed as' ξ

sys4 := [$\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dxdu}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2c\frac{d}{dz}\xi_3 - 2\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc$

$\frac{d^2}{dz^2}\xi_2(y, z) = 0, 2c(t)\frac{\partial^2}{\partial u\partial z}\eta(x, y, z, t, u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y, z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y, z) = 0, a(t)\left(\frac{\partial}{\partial x}\eta(x, y, z, t, u)\right)u + b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x, y, z, t, u)$

sol := *pdsolve* (*sys4*, [$\xi_1, \xi_2, \xi_3, \xi_4, \eta$])

sol := $\left\{ \xi_1 = \int a(t) _C1 dt + _C4, \xi_4 = 0, \eta(x, y, z, t, u) = _C1, \xi_2(y, z) = _C3, \xi_3(y, z) = _C2 \right\}$