$with\left(PDEtools, casesplit, declare\right)$

$$[casesplit, declare]$$

$with\left(DEtools, gensys\right)$

$$[gensys]$$

$$[\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t), a(t), b(t), c(t)]$$

$alias\left(\xi_1 = \xi_1(x,t), \xi_2 = \xi_2(y,z), \xi_3 = \xi_3(y,z), \xi_4 = \xi_4(t), \eta = \eta(x,y,z,t,u), a = a(t), b = b(t), c = c\right.$

$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \eta,\ a,\ b,\ c$$

$declare\left(\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u), a(t), b(t), c(t)\right)$

$\eta(x,y,z,t,u)$ `will now be displayed as` η

$a(t)$ `will now be displayed as` a

$b(t)$ `will now be displayed as` b

$c(t)$ `will now be displayed as` c

$\xi(t)$ `will now be displayed as` ξ

$\xi(x,t)$ `will now be displayed as` ξ

$\xi(y,z)$ `will now be displayed as` ξ

$sys := [\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dx du}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2\,c\frac{d}{dz}\xi_3 - 2\,\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc -$

$\xi_2(y,z) = 0, 2\,c(t)\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y,z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y,z) = 0, a(t)\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u + b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u) -$

$$\frac{\partial^2}{\partial x^2}\xi_1(x,t) = 0$$

$$\frac{\partial^2}{\partial u^2}\eta(x,y,z,t,u) = 0$$

$$\frac{\partial^2}{\partial u\partial x}\eta(x,y,z,t,u) = 0$$

$$c(t)\frac{\partial}{\partial z}\xi_2(y,z) + b(t)\frac{\partial}{\partial y}\xi_3(y,z) = 0$$

$$-\xi_4\frac{d}{dt}c(t) + 2\,c(t)\frac{\partial}{\partial z}\xi_3(y,z) - 2\,\frac{\partial}{\partial x}\xi_1(x,t) = 0$$

$$2\left(\frac{\partial}{\partial z}\xi_3(y,z)\right)b(t)\,c(t) - 2\left(\frac{\partial}{\partial y}\xi_2(y,z)\right)b(t)\,c(t) - b(t)\,\xi_4\frac{d}{dt}c(t) + \xi_4 c(t)\frac{d}{dt}b(t) = 0$$

$$-b(t)\frac{\partial^2}{\partial y^2}\xi_2(y,z) + 2\,b(t)\frac{\partial^2}{\partial u\partial y}\eta(x,y,z,t,u) - c(t)\frac{\partial^2}{\partial z^2}\xi_2(y,z) = 0$$

$$2\,c(t)\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y,z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y,z) = 0$$

$$a(t)\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u+b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u)+c(t)\frac{\partial^3}{\partial x\partial z^2}\eta(x,y,z,t,u)+\frac{\partial}{\partial t}\eta(x,y,z,t,u)+\frac{\partial^3}{\partial x^3}\eta(x,y,z,t,u)=0$$

$$-\xi_4\frac{\mathrm{d}}{\mathrm{d}t}c(t)-c(t)\frac{\mathrm{d}}{\mathrm{d}t}\xi_4(t)+2\,c(t)\frac{\partial}{\partial z}\xi_3(y,z)+c(t)\frac{\partial}{\partial x}\xi_1(x,t)=0$$

$$\eta(x,y,z,t,u)+(c(t))^2\frac{\partial^3}{\partial u\partial z^2}\eta(x,y,z,t,u)-c(t)\frac{\partial}{\partial t}\xi_1(x,t)+2\,a(t)\,c(t)\left(\frac{\partial}{\partial z}\xi_3(y,z)\right)u-u\left(a(t)\frac{\mathrm{d}}{\mathrm{d}t}c(t)-c(t)\frac{\mathrm{d}}{\mathrm{d}t}a(t)\right)\xi_4+\eta(x,y,z$$

$$nops\left([\tfrac{\mathrm{d}^2}{\mathrm{d}x^2}\xi_1=0,\tfrac{\mathrm{d}^2}{\mathrm{d}u^2}\eta=0,\tfrac{\mathrm{d}^2}{\mathrm{d}x\mathrm{d}u}\eta=0,c\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_2+b\tfrac{\mathrm{d}}{\mathrm{d}y}\xi_3=0,-\xi_4\tfrac{\mathrm{d}}{\mathrm{d}t}c+2\,c\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_3-2\,\tfrac{\mathrm{d}}{\mathrm{d}x}\xi_1=0,2\,\left(\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_3\right)bc-\right.$$

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restart

with (*PDEtools*, *casesplit*, *declare*)

[*casesplit*, *declare*]

with (*DEtools*, *gensys*)

[*gensys*]

$$[\xi_1(t),\xi_2(y,z),\xi_3(y,z),\xi_4(t),\eta(x,y,z,t,u)]$$

$$alias(\xi_1=\xi_1(t),\xi_2=\xi_2(y,z),\xi_3=\xi_3(y,z),\xi_4=\xi_4(t),\eta=\eta(x,y,z,t,u))$$

$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \eta$$

$$declare(\xi_1(t),\xi_2(y,z),\xi_3(y,z),\xi_4(t),\eta(x,y,z,t,u))$$

$\eta(x,y,z,t,u)$ `will now be displayed as` η

$\xi(t)$ `will now be displayed as` ξ

$\xi(y,z)$ `will now be displayed as` ξ

$$sys1:=[\tfrac{\mathrm{d}^2}{\mathrm{d}x^2}\xi_1=0,\tfrac{\mathrm{d}^2}{\mathrm{d}u^2}\eta=0,\tfrac{\mathrm{d}^2}{\mathrm{d}x\mathrm{d}u}\eta=0,c\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_2+b\tfrac{\mathrm{d}}{\mathrm{d}y}\xi_3=0,-\xi_4\tfrac{\mathrm{d}}{\mathrm{d}t}c+2\,c\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_3-2\,\tfrac{\mathrm{d}}{\mathrm{d}x}\xi_1=0,2\,\left(\tfrac{\mathrm{d}}{\mathrm{d}z}\xi_3\right)bc$$

$$t,u)-c\frac{\partial^2}{\partial z^2}\xi_2(y,z)=0,2\,c\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u)-b\frac{\partial^2}{\partial y^2}\xi_3(y,z)-c\frac{\partial^2}{\partial z^2}\xi_3(y,z)=0,a\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u+b\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u)+c\frac{\partial^3}{\partial x\partial}$$

$$sol:=pdsolve(sys1,[\xi_1,\xi_2,\xi_3,\xi_4,\eta])$$

$$sol:=\left\{\xi_1=\_C1\,at+\_C6,\xi_4=\_C2,\eta(x,y,z,t,u)=\_C1,\xi_2(y,z)=-\frac{\_C3\,bz}{c}+\_C5,\xi_3(y,z)=\_C3\,y+\_C4\right\}$$

restart

with (*PDEtools*, *casesplit*, *declare*)

[*casesplit*, *declare*]

with (*DEtools*, *gensys*)

[*gensys*]

$$[\xi_1(t),\xi_2(y,z),\xi_3(y,z),\xi_4(t),\eta(x,y,z,t,u),a(t),b(t),c(t)]$$

$$alias(\xi_1=\xi_1(t),\xi_2=\xi_2(y,z),\xi_3=\xi_3(y,z),\xi_4=\xi_4(t),\eta=\eta(x,y,z,t,u),a=a(t),b=b(t),c=c(t$$

$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \eta,\ a,\ b,\ c$$

declare $(\xi_1(t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u), a(t), b(t), c(t))$

$\eta(x,y,z,t,u)$ `will now be displayed as` η

$a(t)$ `will now be displayed as` a

$b(t)$ `will now be displayed as` b

$c(t)$ `will now be displayed as` c

$\xi(t)$ `will now be displayed as` ξ

$\xi(y,z)$ `will now be displayed as` ξ

$sys2 := [\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dxdu}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2c\frac{d}{dz}\xi_3 - 2\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc$

$\xi_2(y,z) = 0, 2c(t)\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y,z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y,z) = 0, a(t)\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u + b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u) +$

sol := *pdsolve* $(sys2, [\xi_1, \xi_2, \xi_3, \xi_4, \eta, a, b, c])$

$\_F5(y,z), \xi_2(y,z) = \xi_2(y,z), \xi_3(y,z) = \xi_3(y,z)\Big\}, \Big[\Big\{\xi_1 = \xi_1, \xi_4 = \xi_4, a(t) = 0, c(t) = 0, \xi_2(y,z) = \int 2\frac{\partial}{\partial u}\eta(x,y,z,t,u)\,dy + \_F2(z)\,y$

restart

with $(PDEtools, casesplit, declare)$

$$[casesplit, declare]$$

with $(DEtools, gensys)$

$$[gensys]$$

$$[\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u)]$$

alias $(\xi_1 = \xi_1(x,t), \xi_2 = \xi_2(y,z), \xi_3 = \xi_3(y,z), \xi_4 = \xi_4(t), \eta = \eta(x,y,z,t,u))$

$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \eta$$

declare $(\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u))$

$\eta(x,y,z,t,u)$ `will now be displayed as` η

$\xi(t)$ `will now be displayed as` ξ

$\xi(x,t)$ `will now be displayed as` ξ

$\xi(y,z)$ `will now be displayed as` ξ

$sys3 := [\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dxdu}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2c\frac{d}{dz}\xi_3 - 2\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc$

$,z,t,u) - c\frac{\partial^2}{\partial z^2}\xi_2(y,z) = 0, 2c\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u) - b\frac{\partial^2}{\partial y^2}\xi_3(y,z) - c\frac{\partial^2}{\partial z^2}\xi_3(y,z) = 0, a\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u + b\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u) + c$

sol := *pdsolve* $(sys3, [\xi_1, \xi_2, \xi_3, \xi_4, \eta])$

$t - 1/2\,\_C1\,xc + \_C7, \xi_4 = -1/2\,\_C1\,(c+2)\,t + \_C3, \eta(x,y,z,t,u) = \_C1\,u + \_C2, \xi_2(y,z) = -1/2\,\_C1\,y - \frac{b\_C4\,z}{c} + \_C6, \xi_3(y,z) = -1/$

$restart$

$with\left(PDEtools, casesplit, declare\right)$

$$[casesplit, declare]$$

$with\left(DEtools, gensys\right)$

$$[gensys]$$

$$[\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u), a(t), b(t), c(t)]$$

$alias\left(\xi_1 = \xi_1(x,t), \xi_2 = \xi_2(y,z), \xi_3 = \xi_3(y,z), \xi_4 = \xi_4(t), \eta = \eta(x,y,z,t,u), a = a(t), b = b(t), c = c\right.$

$$\xi_1,\ \xi_2,\ \xi_3,\ \xi_4,\ \eta,\ a,\ b,\ c$$

$declare\left(\xi_1(x,t), \xi_2(y,z), \xi_3(y,z), \xi_4(t), \eta(x,y,z,t,u), a(t), b(t), c(t)\right)$

$\eta(x,y,z,t,u)$ `will now be displayed as` η

$a(t)$ `will now be displayed as` a

$b(t)$ `will now be displayed as` b

$c(t)$ `will now be displayed as` c

$\xi(t)$ `will now be displayed as` ξ

$\xi(x,t)$ `will now be displayed as` ξ

$\xi(y,z)$ `will now be displayed as` ξ

$sys4 := [\frac{d^2}{dx^2}\xi_1 = 0, \frac{d^2}{du^2}\eta = 0, \frac{d^2}{dx du}\eta = 0, c\frac{d}{dz}\xi_2 + b\frac{d}{dy}\xi_3 = 0, -\xi_4\frac{d}{dt}c + 2c\frac{d}{dz}\xi_3 - 2\frac{d}{dx}\xi_1 = 0, 2\left(\frac{d}{dz}\xi_3\right)bc$

$\frac{\partial^2}{\partial z^2}\xi_2(y,z) = 0, 2c(t)\frac{\partial^2}{\partial u\partial z}\eta(x,y,z,t,u) - b(t)\frac{\partial^2}{\partial y^2}\xi_3(y,z) - c(t)\frac{\partial^2}{\partial z^2}\xi_3(y,z) = 0, a(t)\left(\frac{\partial}{\partial x}\eta(x,y,z,t,u)\right)u + b(t)\frac{\partial^3}{\partial x\partial y^2}\eta(x,y,z,t,u)$

$sol := pdsolve\left(sys4, [\xi_1, \xi_2, \xi_3, \xi_4, \eta]\right)$

$$sol := \left\{\xi_1 = \int a(t)\ \_C1\ dt + \_C4, \xi_4 = 0, \eta(x,y,z,t,u) = \_C1, \xi_2(y,z) = \_C3, \xi_3(y,z) = \_C2\right\}$$