In[16]:= **Integrate[(1+Cos[3*x])^(3/2),x]**

Indefinite integrals: Hide steps

$$\int (1+\cos(3x))^{3/2}\,dx = \frac{1}{18}\left(9\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{9x}{2}\right)\right)(\cos(3x)+1)^{3/2}\sec^3\left(\frac{3x}{2}\right) + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int (\cos(3x)+1)^{3/2}\,dx$$

For the integrand $(\cos(3x)+1)^{3/2}$, substitute $u = 3x$ and $du = 3\,dx$:

$$= \frac{1}{3}\int (\cos(u)+1)^{3/2}\,du$$

Use the half angle identity $\cos(u)+1 = 2\cos^2\left(\frac{u}{2}\right)$:

$$= \frac{1}{3}\int 2\sqrt{2}\,\cos^2\left(\frac{u}{2}\right)^{3/2}\,du$$

Factor out constants:

$$= \frac{2\sqrt{2}}{3}\int \cos^2\left(\frac{u}{2}\right)^{3/2}\,du$$

For the integrand $\cos^2\left(\frac{u}{2}\right)^{3/2}$, substitute $s = \frac{u}{2}$ and $ds = \frac{1}{2}\,du$:

$$= \frac{4\sqrt{2}}{3}\int \cos^2(s)^{3/2}\,ds$$

For the integrand $\cos^2(s)^{3/2}$, simplify powers:

$$= \frac{4\sqrt{2}}{3}\int \cos^3(s)\,ds$$

Use the reduction formula, $\int \cos^m(s)\,ds = \frac{\sin(s)\cos^{m-1}(s)}{m} + \frac{m-1}{m}\int \cos^{-2+m}(s)\,ds$, where $m = 3$:

$$= \frac{4}{9}\sqrt{2}\,\sin(s)\cos^2(s) + \frac{8\sqrt{2}}{9}\int \cos(s)\,ds$$

The integral of $\cos(s)$ is $\sin(s)$:

$$= \frac{8}{9}\sqrt{2}\,\sin(s) + \frac{4}{9}\sqrt{2}\,\sin(s)\cos^2(s) + \text{constant}$$

Substitute back for $s = \frac{u}{2}$:

$$= \frac{8}{9} \sqrt{2} \sin\left(\frac{u}{2}\right) + \frac{1}{9} \sqrt{2} \sin^2(u) \csc\left(\frac{u}{2}\right) + \text{constant}$$

Substitute back for $u = 3x$:

$$= \frac{8}{9} \sqrt{2} \sin\left(\frac{3x}{2}\right) + \frac{1}{9} \sqrt{2} \sin^2(3x) \csc\left(\frac{3x}{2}\right) + \text{constant}$$

Factor the answer a different way:

$$= \frac{2}{9} \sqrt{2} \sin\left(\frac{3x}{2}\right) (\cos(3x) + 5) + \text{constant}$$

Which is equivalent for restricted x values to:

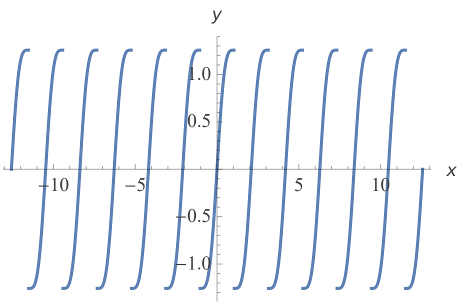
Answer:

$$= \frac{1}{18} \left(9 \sin\left(\frac{3x}{2}\right) + \sin\left(\frac{9x}{2}\right)\right) (\cos(3x) + 1)^{3/2} \sec^3\left(\frac{3x}{2}\right) + \text{constant}$$

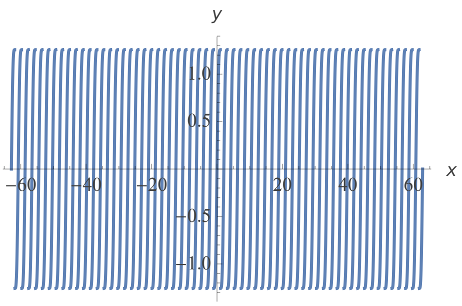
sec(x) is the secant function »

csc(x) is the cosecant function »

Plots of the integral:



min max



min max

Alternate forms of the integral:

More

$$\frac{(1+\cos(3x))^{3/2}\left(9\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{9x}{2}\right)\right)}{18\cos^3\left(\frac{3x}{2}\right)} + \text{constant}$$

$$\frac{(2\cos(x)+1)(\cos(3x)+1)^{3/2}(\cos(3x)+5)\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)}{9(2\cos(x)-1)^3} + \text{constant}$$

$$\left(10\sin\left(\frac{3x}{2}\right)\sqrt{\cos(3x)+1} + 11\sin\left(\frac{9x}{2}\right)\sqrt{\cos(3x)+1} + \sin\left(\frac{15x}{2}\right)\sqrt{\cos(3x)+1}\right) \Big/ \left(9\left(3\cos\left(\frac{3x}{2}\right)+\cos\left(\frac{9x}{2}\right)\right)\right) + \text{constant}$$

Alternate form assuming x>0:

$$\frac{1}{18}\sin\left(\frac{9x}{2}\right)(\cos(3x)+1)^{3/2}\sec^3\left(\frac{3x}{2}\right) + \frac{1}{2}(\cos(3x)+1)^{3/2}\tan\left(\frac{3x}{2}\right)\sec^2\left(\frac{3x}{2}\right) + \text{constant}$$

Expanded form of the integral:

Step-by-step solution

$$\frac{1}{18}\sin\left(\frac{9x}{2}\right)\cos(3x)\sqrt{\cos(3x)+1}\sec^3\left(\frac{3x}{2}\right) + \frac{1}{18}\sin\left(\frac{9x}{2}\right)\sqrt{\cos(3x)+1}\sec^3\left(\frac{3x}{2}\right) + \frac{1}{2}\cos(3x)\sqrt{\cos(3x)+1}\tan\left(\frac{3x}{2}\right)\sec^2\left(\frac{3x}{2}\right) + \frac{1}{2}\sqrt{\cos(3x)+1}\tan\left(\frac{3x}{2}\right)\sec^2\left(\frac{3x}{2}\right) + \text{constant}$$

Series expansion of the integral at x=0:

$$2\sqrt{2}\,x - \frac{9x^3}{2\sqrt{2}} + \frac{567x^5}{160\sqrt{2}} + O\left(x^6\right)$$

(Taylor series)

Big-O notation »

Definite integral (mean square over a period):

More digits

$$\int_0^{\frac{2\pi}{3}} (1+\cos(3x))^3\,dx \approx 5.23599\ldots$$

Definite integral over a half–period:

More digits

$$\int_0^{\frac{\pi}{3}} (1+\cos(3x))^{3/2}\,dx \approx 1.25707872210942\ldots$$

WolframAlpha

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In[17]:= Integrate[(1 + Cos[3 * x]) ^ (3 / 2), x]
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Out[17]= $\frac{1}{18}(1+\text{Cos}[3x])^{3/2}\,\text{Sec}\left[\frac{3x}{2}\right]^3\left(9\,\text{Sin}\left[\frac{3x}{2}\right]+\text{Sin}\left[\frac{9x}{2}\right]\right)$

In[18]:= $\partial_x \left(\frac{1}{18} \left(1 + \text{Cos}[3\,x]\right)^{3/2} \text{Sec}\left[\frac{3\,x}{2}\right]^3 \left(9\,\text{Sin}\left[\frac{3\,x}{2}\right] + \text{Sin}\left[\frac{9\,x}{2}\right]\right)\right)$

Out[18]= $$\frac{1}{18} \left(1 + \text{Cos}[3\,x]\right)^{3/2} \left(\frac{27}{2}\,\text{Cos}\left[\frac{3\,x}{2}\right] + \frac{9}{2}\,\text{Cos}\left[\frac{9\,x}{2}\right]\right) \text{Sec}\left[\frac{3\,x}{2}\right]^3 - \frac{1}{4} \sqrt{1 + \text{Cos}[3\,x]}\; \text{Sec}\left[\frac{3\,x}{2}\right]^3 \text{Sin}[3\,x] \left(9\,\text{Sin}\left[\frac{3\,x}{2}\right] + \text{Sin}\left[\frac{9\,x}{2}\right]\right) + \frac{1}{4} \left(1 + \text{Cos}[3\,x]\right)^{3/2} \text{Sec}\left[\frac{3\,x}{2}\right]^3 \left(9\,\text{Sin}\left[\frac{3\,x}{2}\right] + \text{Sin}\left[\frac{9\,x}{2}\right]\right) \text{Tan}\left[\frac{3\,x}{2}\right]$$

In[19]:= **FullSimplify[%18]**

Out[19]= $\left(1 + \text{Cos}[3\,x]\right)^{3/2}$