Feynman Diagrams
The scattering matrix in coordinates and momentum representation

Mathematical methods for particle physics was one of the weak spots in the Physics package. There existed a FeynmanDiagrams command, but its capabilities were too minimal. People working in the area asked for more functionality. These diagrams are the cornerstone of calculations in particle physics (collisions involving from the electron to the Higgs boson), for example at the CERN. As an introduction for people curious, not working in the area, see "Why Feynman Diagrams are so important".

This post is thus about a new development in Physics: a full rewriting of the FeynmanDiagrams command, now including a myriad of new capabilities (mainly a. b. and c. in the Introduction), reversing the previous status of things entirely. This is work in collaboration with Davide Polvara from Durham University, Centre for Particle Theory.

The complexity of this material is high, so the introduction to the presentation below is as brief as it can get, emphasizing the examples instead. This material is reproducible in Maple 2019.2 after installing the Physics Updates, v.598 or higher.

At the end it is attached the worksheet corresponding to this presentation, as well as the new FeynmanDiagrams help page with all the explanatory details.

Introduction

A scattering matrix $S$ relates the initial and final states, $| i \rangle$ and $| f \rangle$, of an interacting system. In an 4-dimensional spacetime with coordinates $X$, $S$ can be written as:

$$S = T(e^{i \int L(X) dX^4})$$

where $i$ is the imaginary unit and $L$ is the interaction Lagrangian, written in terms of quantum fields depending on the spacetime coordinates $X$. The $T$ symbol means time-ordered. For the terminology used in this page, see for instance chapter IV, "The Scattering Matrix", of ref.[1] Bogoliubov, N.N., and Shirkov, D.V. Quantum Fields.

This exponential can be expanded as

$$S = 1 + S_1 + S_2 + S_3 + ...$$

where

$$S_n = \frac{i^n}{n!} \int \ldots \int T(L(X_1), \ldots, L(X_n)) \, dX_1 \ldots dX_n$$

and $T(L(X_1), \ldots, L(X_n))$ is the time-ordered product of $n$ interaction Lagrangians evaluated at different points. The $S$ matrix formulation is at the core of perturbative approaches in relativistic Quantum Field Theory.

In connection, the FeynmanDiagrams command has been rewritten entirely for Maple 2020. In brief, the new functionality includes computing:

a. The expansion $S = 1 + S_1 + S_2 + S_3 + ...$ in coordinates representation up to arbitrary order (the limitation is now only your hardware)

b. The S-matrix element $\langle f | S | i \rangle$ in momentum representation up to arbitrary order for given number of loops and initial and final particles (the contents of the $| i \rangle$ and $| f \rangle$ states); optionally, also the transition probability density, constructed using the square of the scattering matrix element $|\langle f | S | i \rangle|^2$, as shown in formula (13) of sec. 21.1 of ref.[1].

c. The Feynman diagrams (drawings) related to the different terms of the expansion of $S$ or of its matrix elements $\langle f | S | i \rangle$.

Interaction Lagrangians involving derivatives of fields, typically appearing in non-Abelian gauge theories, are also handled, and several options are provided enabling restricting the outcome in different ways, regarding the incoming and outgoing particles, the number of loops, vertices or external legs, the propagators and normal products, or whether to compute tadpoles and 1-particle reducible terms.
**Examples**

For illustration purposes set three coordinate systems, and set $\phi$ to represent a quantum operator

> with(Physics):

> Setup(mathematicalnotation = true, coordinates = \{X, Y, Z\}, quantumoperators = \{\phi\})

Systems of spacetime coordinates are: \(X = (x1, x2, x3, x4), Y = (y1, y2, y3, y4), Z = (z1, z2, z3, z4)\)

\[
\text{coordinatesystems} = \{X, Y, Z\}, \text{mathematicalnotation} = \text{true}, \text{quantumoperators} = \{\phi\}
\] (1.1)

Let $L$ be the interaction Lagrangian

> \[ L := \lambda \phi(X)^4 \]

\[ L := \lambda \phi(X)^4 \] (1.2)

The expansion of $S$ in coordinates representation, computed by default up to order $= 3$ (you can change that using the option order $= n$), by definition containing all possible configurations of external legs, displaying the related Feynman Diagrams, is given by

> \[ S_{\text{order}} = \text{FeynmanDiagrams}(L, \text{diagrams}) \]

\[
S_{\text{order}} = 1 + \int \lambda : \phi(X)^4 : dX^4 + \frac{1}{2!} \int 16 \lambda^2 : \phi(X)^3 \phi(Y)^3 : \{\phi(X), \phi(Y)\}
\] (1.3)
The expansion of $S$ in coordinates representation to a specific order shows in a compact way the topology of the underlying Feynman diagrams. Each integral is represented with a new command, `FeynmanIntegral`, that works both in coordinates and momentum representation. To each term of the integrands corresponds a diagram, and the correspondence is always clear from the symmetry factors.

In a typical situation, one wants to compute a specific term, or scattering process, instead of the $S$ matrix up to some order with all possible configurations of external legs. For example, to compute only the terms of this result that correspond to diagrams with 1 loop use `numerosloops = 1` (for tree-level, use `numerosloops = 0`)

$$S_{\text{order} = 3, \text{loops} = 1} = \text{FeynmanDiagrams}(L, \text{numerosloops} = 1, \text{diagrams})$$

In the result above there are two terms, with 4 and 6 external legs respectively.

A scattering process with matrix element $\langle f \mid S \mid i \rangle$ in momentum representation, corresponding to the term with 4 external legs (symmetry factor = 72), could be any process where the total number of incoming + outgoing parties is equal to 4. For example, one with 2 incoming and 2 outgoing particles. The transition probability for that process is given by

$$\langle \phi, \phi \mid S \mid \phi, \phi \rangle = \text{FeynmanDiagrams}(L, \text{incomingparticles} = [\phi, \phi], \text{outgoingparticles} = [\phi, \phi], \text{numerosloops} = 1, \text{diagrams})$$

$$\langle \phi, \phi \mid S \mid \phi, \phi \rangle = \frac{9 \lambda^2 \delta(\mathbf{p}_3 - \mathbf{p}_4 + \mathbf{p}_1 + \mathbf{p}_2)}{8 \pi \sqrt{E_1 E_2 E_3 E_4} (p_2^2 - m_2^2 + \epsilon) \left( (-\mathbf{p}_1 - \mathbf{p}_2)^2 - m_2^2 + \epsilon \right)} \left( \frac{1}{p_2^4} + \text{terms} \right)$$

(1.5)
When computing in momentum representation, only the topology of the corresponding Feynman diagrams is shown (i.e. the diagrams associated to the corresponding Feynman integral in coordinates representation).

The transition matrix element \( \langle f | S | i \rangle \) is related to the transition probability density \( dw \) (formula (13) of sec. 21.1 of ref. [1]) by

\[
dw = (2 \pi)^3 s^{-4} n_1 ... n_s |F(p_i, p_f)|^2 \delta \left( \sum_{i=1}^{s} p_i - \sum_{f=1}^{r} p_f \right) d^3 p_1 ... d^3 p_r
\]

where \( n_1 ... n_s \) represent the particle densities of each of the \( s \) particles in the initial state \( |i\rangle \), the \( \delta \) (Dirac) is the expected singular factor due to the conservation of the energy-momentum and the amplitude \( F(p_i, p_f) \) is related to \( \langle f | S | i \rangle \) via

\[
\langle f | S | i \rangle = F(p_i, p_f) \delta \left( \sum_{i=1}^{s} p_i - \sum_{f=1}^{r} p_f \right)
\]

To directly get the probability density \( dw \) instead of \( \langle f | S | i \rangle \) use the option \( output = probabilitydensity \)

\[
4 \pi^2 \prod_{i=1}^{s} n_i |F|^2 \delta \left( -P_3 - P_4 + P_1 + P_2 \right) \prod_{f=1}^{r} dP_f \text{ where } F =
\]

In practice, the most common computations involve processes with 2 or 4 external legs. To restrict the expansion of the scattering matrix in coordinates representation to that kind of processes use the \( numberofexternallegs \) option. For example, the following computes the expansion of \( S \) up to \( order = 3 \), restricting the outcome to the terms corresponding to diagrams with only 2 external legs

\[ S \bigg| \text{[order = 3, legs = 2]} \bigg] = \text{FeynmanDiagrams}(L, \text{numberofexternallegs} = 2, \text{diagrams}) \]
This computation can also be performed to higher orders. For example, with 3 loops, in coordinates, and momentum representation for a process related to the first integral (1 term with symmetry factor = 96) is then

$$S_{[\text{order } = 3, \text{ legs } = 2]} = \frac{P^2}{2T} \int \int 96 \lambda^2 : \phi(X) \phi(Y) : [\phi(X), \phi(Y)]^3 dX^4 dY^4 + \frac{P^3}{3T} \int \int 3456 \lambda^3 : \phi(X)^2 : [\phi(X), \phi(Y)]^3 + 10368 \lambda^3 : \phi(X) \phi(Y) : [\phi(X), \phi(Y)] [\phi(X), \phi(Z)]^2 \phi(Y), \phi(Z)^2 dX^4 dY^4 dZ^4$$

This result shows two Feynman integrals, with respectively 2 and 3 loops, the second integral with two terms. The transition probability density in momentum representation for a process related to the first integral (1 term with symmetry factor = 96) is then

> **FeynmanDiagrams**

\(L, \text{ incoming particles } = [\phi], \text{ outgoing particles } = [\phi], \text{ number of loops } = 2, \text{ diagrams, output } = \text{ probability density} \)

$$\frac{1}{2\pi} \prod_{i=1}^{n} \left| F \right|^2 \delta(-P_2 + P_1) \prod_{j=1}^{1} dP^{2}_{j} \text{ where } F = \int \left( \frac{3^2 \lambda^2}{8} \frac{\pi^2}{\sqrt{E_1 E_2}} \right) \left( p_{2}^2 - m_{\phi}^2 + 1 \epsilon \right) \left( p_{3}^2 - m_{\phi}^2 + 1 \epsilon \right) \left( (P_1 - P_2 - P_3)^2 - m_{\phi}^2 + 1 \epsilon \right) (\frac{p_2^4}{3} \frac{p_3^4}{3})$$

In the above, for readability, the contracted spacetime indices in the square of momenta entering the amplitude \(F\) (as denominators of propagators) are implicit. To make those indices explicit, use the option *putindicesinsquareofmomentum*

> **FeynmanDiagrams**

\(L, \text{ incoming } = [\phi], \text{ outgoing } = [\phi], \text{ number of loops } = 2, \text{ indices} \)

\* Partial match of 'indices' against keyword 'putindicesinsquareofmomentum'

\(F = \left( \frac{3^2 \lambda^2}{8} \frac{\pi^2}{\sqrt{E_1 E_2}} \right) \left( p_{2}^2 - m_{\phi}^2 + 1 \epsilon \right) \left( p_{3}^2 - m_{\phi}^2 + 1 \epsilon \right) \left( (P_1 - P_2 - P_3)^2 - m_{\phi}^2 + 1 \epsilon \right) (\frac{p_2^4}{3} \frac{p_3^4}{3}) \)

This computation can also be performed to higher orders. For example, with 3 loops, in coordinates and momentum representations, corresponding to the other two terms and diagrams in ??

> **S_3**

\(\text{[legs = 2, loops = 3]} \)

\* Partial match of 'legs' against keyword 'numberoflegs'

\* Partial match of 'loops' against keyword 'numberofloops'

$$S_3 = \left( \frac{P^3}{3T} \int \int 3456 \lambda^3 : \phi(X)^2 : [\phi(X), \phi(Y)]^3 + 10368 \lambda^3 : \phi(X) \phi(Y) : [\phi(X), \phi(Y)] [\phi(X), \phi(Z)]^2 \phi(Y), \phi(Z)^2 dX^4 dY^4 dZ^4$$

A corresponding S-matrix element in momentum representation:

> (\phi | S_3 | \phi) \text{ for loops } = 3

$$= \text{FeynmanDiagrams}(L, \text{ incoming particles } = [\phi], \text{ outgoing particles } = [\phi], \text{ number of loops } = 3)$$
Consider the interaction Lagrangian of Quantum Electrodynamics (QED). To formula this problem on the worksheet, start defining the vector field $A$. 

> Define($A$)

Defined objects with tensor properties

\[
\{A, \gamma^\mu, P_\mu, P_\mu^\nu, \alpha^\mu \in \mathbb{D}, g, \phi, \psi, P_\mu, P_\mu^\nu, p_\mu, p_\mu^\nu, \epsilon, \alpha, \beta, \mu, \nu, X, Y, Z\} \tag{1.12}
\]

Set lowercase Latin letters from $i$ to $s$ to represent spinor indices (you can change this setting according to your preference, see Setup), also the (anticommutative) spinor field will be represented by $\psi$, so set $\psi$ as an anticommutative prefix, and set $A$ and $\psi$ as quantum operators

> Setup(spinnerindices = lowercaselatin_is, anticommutativeprefix = $\psi$, op = \{A, $\psi$\})

* Partial match of 'op' against keyword 'quantumoperators'*

\[
\text{anticommutativeprefix} = \{\psi\}, \text{quantumoperators} = \{A, \phi, \psi\}, \text{spinnerindices} = \text{lowercaselatin_is} \tag{1.13}
\]

The matrix indices of the Dirac matrices are written explicitly and use conjugate to represent the Dirac conjugate $\psi_i^\dagger$.

> L_{\text{QED}} := \alpha \text{conjugate}(\psi[j](X)) Dgamma[\mu][j, k] \psi[k](X) A[\mu](X) \tag{1.14}

L_{\text{QED}} := \alpha \psi[j](X) \psi[k](X) A[\mu](X) \gamma^\mu_{j,k}

Compute $S_2$, only the terms with 4 external legs, and display the diagrams: all the corresponding graphs have no loops

> $S_2 \mid_{\text{legs} = 4} = \text{FeynmanDiagrams}(L_{\text{QED}}, \text{numberofvertices} = 2, \text{numberoflegs} = 4, \text{diagrams})$

\[
\begin{align*}
S_2 \mid_{\text{legs} = 4} &= \frac{12}{2!} \int_{-2}^{2} |2\alpha^2 \gamma_{j,k}^\mu \gamma_{i,l}^\alpha : \psi_k(X) A[\mu](X) \psi_i^\dagger(Y) A[\alpha](Y) : \psi_j(Y), \psi_j(Y)| \tag{1.15}
\end{align*}
\]
\[
+ \alpha^2 \left( \gamma^\mu \right)_{j,k} \left( \gamma^\alpha \right)_{l,i} \cdot \psi_j(X) \psi_k(X) \psi_l(Y) \psi_i(Y) \cdot \left[ A_{\mu}(X), \ A_{\alpha}(Y) \right] \ dX^4 dY^4
\]

The same computation but with only 2 external legs results in the diagrams with 1 loop that correspond to the self-energy of the electron and the photon (page 218 of ref.[1])

\[
\left[ S\right] = \text{FeynmanDiagrams} \left( L_{\text{QED}}, \ \text{numberofvertices} = 2, \ \text{numberoflegs} = 2, \ \text{diagrams} \right)
\]

\[
\int \frac{d^4 X}{2!} \left[ -2 \alpha^2 \left( \gamma^\mu \right)_{j,k} \left( \gamma^\alpha \right)_{l,i} \cdot \psi_j(X) \psi_k(X) \psi_l(Y) \psi_i(Y) \cdot \left[ A_{\mu}(X), \ A_{\alpha}(Y) \right] \ dX^4 dY^4 \right]
\]

\[
\int \frac{d^4 X}{2!} \left[ \alpha^2 \left( \gamma^\mu \right)_{j,k} \left( \gamma^\alpha \right)_{l,i} \cdot \psi_j(Y) \psi_k(Y) \psi_l(X) \psi_i(X) \cdot \left[ \bar{A}_{\mu}(Y), \ \bar{A}_{\alpha}(X) \right] \ dX^4 dY^4 \right]
\]

where the diagram with two spinor legs is the electron self-energy. To restrict the output furthermore, for example getting only the self-energy of the photon, you can specify the normal products you want:

\[
\left[ S\right] = \text{FeynmanDiagrams} \left( L_{\text{QED}}, \ \text{numberofvertices} = 2, \ \text{numberoflegs} = 2, \ \text{normalproduct} \right)
\]

\[
\int \frac{d^4 X}{2!} \left[ A^2 \cdot \right]
\]

* Partial match of 'normalproduct' against keyword 'normalproducts'

\[
\int \frac{d^4 X}{2!} \left[ A^2 \cdot \right]
\]

The corresponding S-matrix elements in momentum representation

\[
\left\{ \psi | S | \psi \right\} = \text{FeynmanDiagrams} \left( L_{\text{QED}}, \ \text{incomingparticles} = [\psi], \ \text{outgoing} = [\psi], \ \text{numberofloops} = 1, \ \text{diagrams} \right)
\]

\[
\left\{ \psi | S | \psi \right\} = - \left[ \frac{1}{8 \pi^3 \left( P_1^2 \cdot m^2 + 1 \epsilon \right) \left( P_2^2 \cdot m^2 + 1 \epsilon \right)} \left( u^\dagger \psi \right) \left( P_1^\dagger \psi \right) \left( u^\dagger \psi \right) \left( P_2^\dagger \psi \right) \right] g_{\alpha \nu}
\]
\[ \left( 1 \mp \frac{P_2 \cdot P_1}{m_A^2} \right) \left( \frac{P_1}{m_A} \gamma^\mu \right)_{\alpha \beta} \left( \frac{P_2}{m_A} \gamma^\nu \right)_{\beta \gamma} \left( P_{1 \beta} + P_{2 \beta} \right) \left( \gamma^\beta \right)_{m,n} + m \delta_{m,n} \delta(-P_2 + P_1) \right) dp_2^4 \]

In this result we see \( u \) spinor (see ref.[2]), and the propagator of the field \( A_\mu \) with a mass \( m_A \). To indicate that this field is massless use

\[ \text{Setup}(\text{massless} = A) \]

\[ \text{masslessfields} = (A) \]

Now the propagator for \( A_\mu \) is the one of a massless vector field:

\[ \text{FeynmanDiagrams}(L_{\text{QED}}, \text{incoming} = [\psi], \text{outgoing} = [\psi], \text{numberofloops} = 1) \]

\[ - \left( \left( u_\alpha \right)_I \left( \gamma_\mu \right)_I \right) \frac{2 \alpha}{g} \frac{\epsilon_{\alpha \beta \gamma \delta}}{2} \left( P_{1 \beta} \left( \gamma^\beta \right)_{m,n} + m \delta_{m,n} \right) \delta(-P_2 + P_1) \]

\[ \frac{8 \pi^3}{(P_1 + P_2)^2} \left( p_2^2 + 1 \epsilon \right) \left( (P_1 + P_2)^2 - m_\psi^2 + 1 \epsilon \right) \]

\[ \frac{4 \pi^3}{(P_1 + P_2)^2} \left( p_2^2 + 1 \epsilon \right) \left( (P_1 + P_2)^2 - m_\psi^2 + 1 \epsilon \right) \]

The self-energy of the photon:

\[ \langle A \mid S \mid A \rangle = \text{FeynmanDiagrams}(L_{\text{QED}}, \text{incomingparticles} = [A], \text{outgoing} = [A], \text{numberofloops} = 1) \]

\[ \langle A \mid S \mid A \rangle = - \left( \left( \epsilon_\alpha \left( \gamma_\mu \right) \left( \gamma_\nu \right) m \delta_{l,n} + p_{2 \nu} \left( \gamma^\beta \right) m \delta_{l,n} \right) \frac{\epsilon_{\alpha \beta \gamma \delta}}{2} \left( P_{1 \beta} \left( \gamma^\beta \right)_{m,n} + m \delta_{m,n} \right) \delta(-P_2 + P_1) \right) \left( 16 \pi^3 \sqrt{E_1 E_2} (p_2^2 - m_\psi^2 + 1 \epsilon) \left( (P_1 + P_2)^2 - m_\psi^2 + 1 \epsilon \right) \right) \]

\[ \frac{4 \pi^3}{(P_1 + P_2)^2} \left( p_2^2 + 1 \epsilon \right) \left( (P_1 + P_2)^2 - m_\psi^2 + 1 \epsilon \right) \]

where \( \epsilon_\alpha \) is the corresponding polarization vector.

When working with non-Abelian gauge fields, the interaction Lagrangian involves derivatives. \text{FeynmanDiagrams} can handle that kind of interaction in \textit{momentum representation}. Consider for instance a Yang-Mills theory with a massless field \( B_a \) where \( a \) is a SU2 index (see eq.(12) of sec. 19.4 of ref.[1]). The interaction Lagrangian can be entered as follows

\[ \text{Setup}(\text{su2indices} = \text{lowercaselatin}_ah, \text{massless} = B, \text{op} = B) \]

\[ \text{masslessfields} = (A,B), \text{quantumoperators} = \{A,B,\phi,\psi,\psi \} \]

\[ \text{su2indices} = \text{lowercaselatin}_ah \]

\[ \text{Define}(B[\mu, a], \text{quiet}) : \]

\[ F_B[\mu, \nu, a] := d_{\nu} [B[\mu, a]](X) - d_{\mu} [B[\nu, a]](X) \]

\[ F_B[\mu, \nu, a] := \partial_{\nu} (B[\mu, a](X)) - \partial_{\mu} (B[\nu, a](X)) \]

\[ L := \frac{g}{2} \text{LeviCovita}[a, b, c] F_B[\mu, \nu, a] B[\mu, b](X) B[\nu, c](X) + \frac{g^2}{4} \text{LeviCovita}[a, b, c] \text{LeviCovita}[a, e, f] B[\mu, b](X) B[\nu, e](X) B[\nu, f](X) \]

\[ L := \frac{g}{2} \epsilon_{a, b, c} \left( \partial_{\mu} (B[\nu, a](X)) - \partial_{\nu} (B[\mu, a](X)) \right) B[\mu, b](X) B[\nu, c](X) \]

\[ + \frac{g^2}{4} \epsilon_{a, b, c} \left( \partial_{\mu} (B[\nu, a](X)) - \partial_{\nu} (B[\mu, a](X)) \right) B[\mu, e](X) B[\nu, f](X) \]

The transition probability density at tree-level for a process with two incoming and two outgoing \( B \) particles is given by
\[ \left( -P_3 \sigma - P_4 \sigma + P_1 \sigma + P_2 \sigma \right) \]
\[ + P_4 \sigma g^{\alpha, \lambda} + \left( P_I^{\alpha} - 2 P_4^{\alpha} \right) g^{\lambda, \sigma} - 2 g^{\alpha, \sigma} \left( P_I^{\lambda} - \frac{P_4^{\lambda}}{2} \right) \epsilon_{a_1, a_2, d} \alpha, \tau \delta_{a_2, a_3} \]

\[ - \frac{1}{16} \left[ \delta_{g, h} g^{\alpha, \beta} g^{\kappa, \lambda} + g^{\alpha, \kappa} g^{\beta, \lambda} - 2 g^{\beta, \kappa} g^{\alpha, \lambda} \right] + \delta_{d, h} \left( g^{\alpha, \beta} g^{\kappa, \lambda} - 2 \right) g^{\alpha, \kappa} g^{\beta, \lambda} \]

To simplify the repeated indices, use the option \textit{simplifytensorindices}. To check the indices entering a result like this one use \textit{Check}; there are no free indices, and regarding the repeated indices:

\[ \check{1.26}, \text{all} \]

The repeated indices per term are: \{(\ldots), (\ldots), \ldots\} and the free indices are: \{(\ldots)\}

\[ \{a_1, a_2, a_3, \alpha, \beta, \chi, d, g, h, \kappa, \lambda, \sigma, \tau\}, \emptyset \]

(1.27)

This process can be computed with 1 or more loops, in which case the number of terms increases significantly. As another interesting non-Abelian model, consider the interaction Lagrangian of the electro-weak part of the Standard Model

\[ > \text{Coordinates(clear, } Z) \]

Unaliasing \(Z\) previously defined as a system of spacetime coordinates

\[ > \text{Setup(quantumoperators } = \{ W, Z \}) \]

\[ \text{quantumoperators} = \{ A, B, W, Z, \phi, \psi, \varphi I \} \]

(1.29)

\[ > \text{Define}(W[\mu], Z[\mu]) \]

Defined objects with tensor properties

\[ \{ A, B, \alpha, \beta, \gamma, \mu, \nu, \rho, \sigma, P_1, P_2, P_3, P_4, \alpha, \beta, \gamma, \mu, \nu, \rho, \sigma, p_1, p_2, p_3, p_4, p_5, \psi, \epsilon, P, X, Y \} \]

(1.30)

\[ > \text{CompactDisplay}(W[\mu], Z[\mu]) \]

\[ W(X) \text{ will now be displayed as } W \]

\[ Z(X) \text{ will now be displayed as } Z \]

(1.31)

\[ F_W[\mu, v] := d_{[\mu]}(W[v](X)) - d_{[v]}(W[\mu](X)) \]

\[ F_{W[\mu, v]} := \partial_{\mu} (W[v]) - \partial_{\nu} (W[\mu]) \]

(1.32)

\[ F_Z[\mu, v] := d_{[\mu]}(Z[v](X)) - d_{[v]}(Z[\mu](X)) \]

\[ F_{Z[\mu, v]} := \partial_{\mu} (Z[v]) - \partial_{\nu} (Z[\mu]) \]

(1.33)

\[ L_{WZ} := Ig \cos(\theta_w) \left( \left( \text{Dagger}(F_W, \mu, v) \right) W[\mu](X) - \text{Dagger}(W[\mu](X)) F_{W[\mu, v]} \right) Z[v](X) \]

\[ + W[v](X) \text{Dagger}(W[\mu](X)) F_{Z[\mu, v]} \]

\[ L_{WZ} := Ig \cos(\theta_w) \left( \left( \partial_{\mu} (W[v]^-) - \partial_{\nu} (W[v]^+) \right) W[\mu] - W[\mu] \left( \partial_{\mu} (W[v]) - \partial_{\nu} (W[\mu]) \right) \right) Z[v] + W[v] W[\mu] \left( \partial_{\mu} (Z[v]) \right) \]  \[ - \partial_{\nu} (Z[\mu]) \]

(1.34)

This interaction Lagrangian contains six different terms. The S-matrix element for the tree-level process with two incoming and two outgoing $W$ particles is shown in the help page for \textit{FeynmanDiagrams}.

\[ > \]

\textbf{References}
