$$q_x = -\kappa \frac{\partial T}{\partial x} \tag{1}$$

$$q_y = -\kappa \frac{\partial T}{\partial y} \tag{2}$$

$$q_z + \tau_q \frac{\partial q_z}{\partial t} = -k\left(\frac{\partial T}{\partial z} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial z}\right)\right) \tag{3}$$

On the other hand, by putting the equations $q_x = -\kappa \frac{\partial T}{\partial x}$ and $q_y = -\kappa \frac{\partial T}{\partial y}$ in the equation $-\nabla . q + Q = \rho c_p \frac{\partial T}{\partial t}$ we have:

$$\frac{\partial q_z}{\partial z} = -\rho c_p \frac{\partial T}{\partial t} + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q \tag{4}$$

By taking derivative from the equation $q_z + \tau_q \frac{\partial q_z}{\partial t} = -k\left(\frac{\partial T}{\partial z} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial z}\right)\right)$ concerning the z:

$$\frac{\partial q_z}{\partial z} + \tau_q \frac{\partial^2 q_z}{\partial t \partial z} = -k\left(\frac{\partial^2 T}{\partial z^2} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)\right) \tag{5}$$

By replacing the relationship $\frac{\partial q_z}{\partial z} = -\rho c_p \frac{\partial T}{\partial t} + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q$ in the equation, $\frac{\partial q_z}{\partial z} + \tau_q \frac{\partial^2 q_z}{\partial t \partial z} = -k\left(\frac{\partial^2 T}{\partial z^2} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)\right)$ the thin film heat transfer equation for dual phase lag is obtained as follow:

$$k\left(-\frac{\rho c_p}{k}\frac{\partial T}{\partial t} + \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{Q}{k}\right) + \tau_q k\left(-\frac{\rho c_p}{k}\frac{\partial^2 T}{\partial t^2} + \left(\frac{\partial^3 T}{\partial t \partial x^2} + \frac{\partial^3 T}{\partial t \partial y^2}\right) + \frac{\frac{\partial Q}{\partial t}}{k}\right) = -k\left(\frac{\partial^2 T}{\partial z^2} + \tau_T \frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)\right) \tag{6}$$

And by simplification we have

$$\left(-\frac{\rho c_p}{k}\frac{\partial T}{\partial t}+\left(\nabla^2 T\right)+\frac{Q}{k}\right)+\tau_q\left(-\frac{\rho c_p}{k}\frac{\partial^2 T}{\partial t^2}+\left(\frac{\partial^3 T}{\partial t\partial x^2}+\frac{\partial^3 T}{\partial t\partial y^2}\right)+\frac{\frac{\partial Q}{\partial t}}{k}\right)= -\left(\frac{\partial^2 T}{\partial z^2}+\tau_T\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)\right) \tag{7}$$

$$-\frac{\rho c_p}{k}\frac{\partial T}{\partial t}+\nabla^2 T+\frac{Q}{k}+\tau_q\left(\frac{\partial^3 T}{\partial t\partial x^2}+\frac{\partial^3 T}{\partial t\partial y^2}\right)-\tau_q\frac{\rho c_p}{k}\frac{\partial^2 T}{\partial t^2}+\tau_q\frac{\frac{\partial Q}{\partial t}}{k}= -\left(\frac{\partial^2 T}{\partial z^2}+\tau_T\frac{\partial}{\partial t}\left(\frac{\partial^2 T}{\partial z^2}\right)\right) \tag{8}$$

$$\nabla^2 T+\frac{Q}{k}+\tau_q\left(\frac{\partial^3 T}{\partial t\partial x^2}+\frac{\partial^3 T}{\partial t\partial y^2}\right)+\tau_q\frac{\frac{\partial Q}{\partial t}}{k}+\frac{\partial^2 T}{\partial z^2}+\tau_T\frac{\partial^3 T}{\partial t\partial z^2}=\frac{\rho c_p}{k}\left(\frac{\partial T}{\partial t}+\tau_q\frac{\partial^2 T}{\partial t^2}\right) \tag{9}$$

$$\nabla^2 T+\frac{Q+\tau_q\frac{\partial Q}{\partial t}}{k}+\tau_q\left(\frac{\partial^3 T}{\partial t\partial x^2}+\frac{\partial^3 T}{\partial t\partial y^2}\right)+\tau_T\frac{\partial^3 T}{\partial t\partial z^2}=\frac{\rho c_p}{k}\left(\frac{\partial T}{\partial t}+\tau_q\frac{\partial^2 T}{\partial t^2}\right) \tag{10}$$