Your problem very much looks like a constraint mechanical system with stiffness matrix K_{11} and mass matrix M_{11} for the corresponding free system and constraint matrix K_{12}'. For that reason I assume here that $N < M$ and K_{12} has rank N.

Let $K_{12} = Q\begin{pmatrix}R\\0\end{pmatrix}$ be a QR decomposition of K_{12}. We use $\begin{pmatrix}Q & 0\\0 & 1\end{pmatrix}$ as transformation matrix for your eigenproblem.

Your stiffness matrix then transforms like $\begin{pmatrix}Q' & 0\\0 & 1\end{pmatrix}\begin{pmatrix}K_{11} & K_{12}\\K_{12}' & 0\end{pmatrix}\begin{pmatrix}Q & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}Q'K_{11}Q & \begin{pmatrix}R\\0\end{pmatrix}\\ \begin{pmatrix}R' & 0\end{pmatrix} & 0\end{pmatrix}$.

We name the left-upper part of the transformed stiffness and mass matrix as $\bar{K} = Q'K_{11}Q$, $\bar{M} = Q'M_{11}Q$ and the transformed coordinates as $\bar{X}$. The Lagrange coefficients λ retain in the transformed system.

Because of the last N transformed equations $R\cdot\bar{X}(1:N) = 0$ the first N components of $\bar{X}$ are zero. Thus we can just split off those components from $\bar{X}$ and only consider $X(N+1:M)$ in the following. Furthermore, the fist N transformed equations $\left(\bar{K}(1:N,N+1:M) - \omega^2\bar{M}(1:N,N+1:M)\right)\cdot\bar{X}(N+1:M) + R\cdot\lambda = 0$ just determine λ if ω^2 and $\bar{X}(N+1:M)$ are known: $\lambda = -R^{-1}\cdot\left(\bar{K}(1:N,N+1:M) - \omega^2\bar{M}(1:N,N+1:M)\right)\cdot\bar{X}(N+1:M)$ They are irrelevant for the actual eigenproblem in transformed coordinates.

The only transformed equations relevant for the eigenproblem are:
$\left(\bar{K}(N+1:M,N+1:M) - \omega^2\bar{M}(N+1:M,N+1:M)\right)\bar{X}(N+1:M) = 0$

Thus you only need to solve the reduced eigenproblem with the reduced positive definite system matrices $\bar{K}(N+1:M,N+1:M) = Q_{\mathrm{pr}}'K_{11}Q_{\mathrm{pr}}$ and $\bar{M}(N+1:M,N+1:M) = Q_{\mathrm{pr}}'M_{11}Q_{\mathrm{pr}}$ where $Q_{\mathrm{pr}} = Q(:,N+1:M)$.