Helicopter Math Model

FWK 7/12/2022

restarted using what I've learned

$tm := \text{Vector}(10, 0, \text{orientation} = \text{row})$ $tm(1) := \text{time}()$

Basic Parameters

Blades rectangular Number of blades $N_{bld} := 4$

$D_{rtr} := 30 ft$ $R_{tip} := \frac{D_{rtr}}{2} = 15\ ft$ Blade chord $chd := 8 in$ $R_{hng} := 1 ft$

Solidity $\sigma := \frac{N_{bld} \cdot chd}{\pi \cdot R_{tip}} = 0.057$

Air density $\rho_{air} = 0.076\ \frac{lb}{ft^3}$ Speed of sound $a_{air} = 1116.324\ \frac{ft}{s}$

If the tip Mach Number is Then the rotation speed is

$M_{tip} := 0.62$ $\omega := \frac{M_{tip} \cdot a_{air}}{R_{tip}} = 440.6\ rpm$

$tm(2) := \text{time}()$

NACA 0012 lift and drag

lift

drag

$Cl_{\alpha} := \frac{0.1}{1 deg}$ $C_L := \alpha \rightarrow \alpha \cdot Cl_{\alpha}$ $C_L(10 deg) = 1.000$

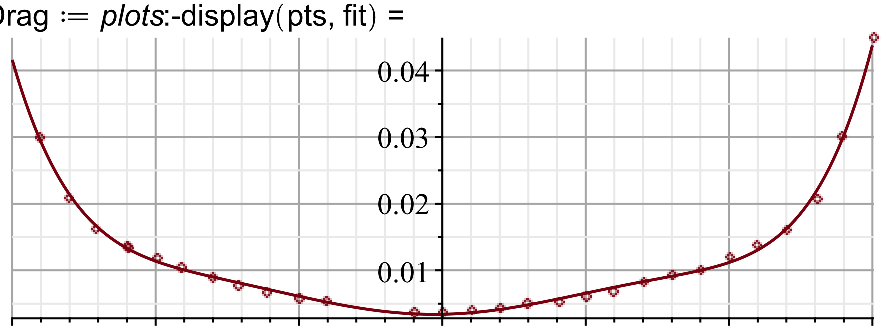
data := Import("C:/Users/fkohl/Desktop/excess mcad/NACA0012 CD alpha.csv")

Data := convert(data,'Matrix') alp := Column(Data, 1) · 1deg cd := Column(Data, 2)

ft := *Statistics*:-PolynomialFit(6, Data, x) $C_D := \alpha \rightarrow \text{eval}(ft, x = \alpha)$

pts := *plot*(Data, style = point) fit := plot($C_D(z)$, z = −15 ..15, gridlines)

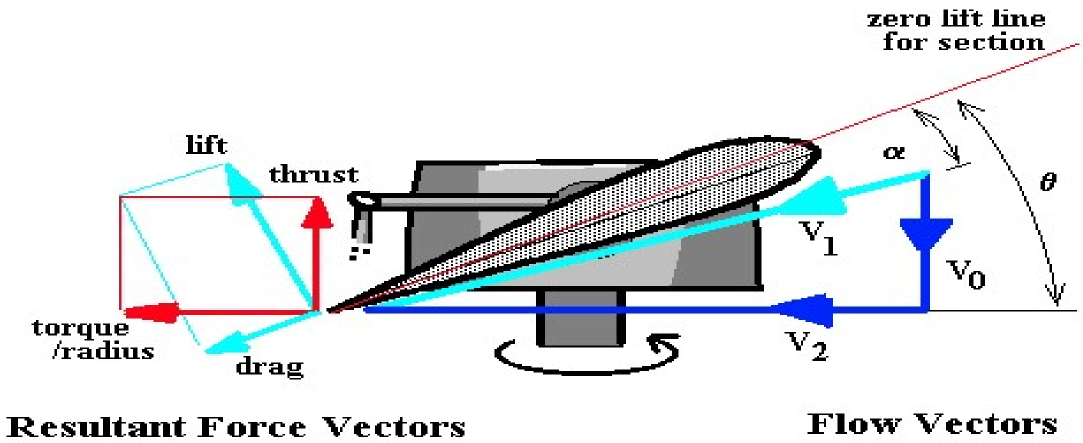
Drag := *plots*:-display(pts, fit) =



−15 −10 −5 0 5 10 15

$tm(3) := time()$

Physics and Discussion



$V_2 = \omega \cdot r$

$V_0 = u$

$\alpha = \arctan\left(\frac{u}{\omega \cdot r}\right)$

The lift available from an airfoil depends on three factors: the angle of the airfoil into the relative velocity, the magnitude of the relative velocity, and the density of the air. $L = \frac{C_L(\alpha) \cdot \rho \cdot V^2 \cdot Area}{2}$

The velocity past the local airfoil is predominantly due to the rotation of the blade, so we increase the angle of the blade at the inboard end to offset the reduced velocity.

$$\alpha_i := (r, \theta, u) \rightarrow \theta - \arctan\left(\frac{u}{\omega \cdot r}\right) - 8deg \cdot \frac{r}{R_{tip}} + 6deg$$

The basic parameters of a rotor blade section are shown here. The blade is an airfoil; lift and drag are oriented to the local velocity vector while thrust and torque force are oriented to the axis of rotation.

Lift and drag of a local section:

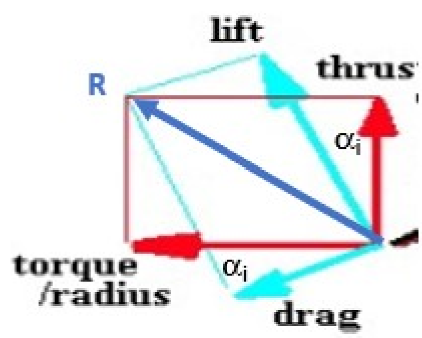
$$lf_s := (r, \theta, u) \rightarrow \frac{\rho_{air} \cdot chd \cdot (\omega^2 \cdot r^2 + u^2)}{2} \cdot C_L(\alpha_i(r, \theta, u))$$

Functions for lift and drag coefficients were developed for the NACA 0012 airfoil above.

$$dr_s := (r, \theta, u) \rightarrow \frac{\rho_{air} \cdot chd \cdot (\omega^2 \cdot r^2 + u^2)}{2} \cdot C_D(\alpha_i(r, \theta, u))$$

Thrust and torque of a local section:

Lift and drag are relative to the local wind vector, we need the forces oriented to the rotor shaft.



$$Rot := \begin{bmatrix} CC & -SS \\ SS & CC \end{bmatrix} \quad LD := \begin{bmatrix} Ll \\ Dd \end{bmatrix}$$

$T := L \cdot \cos - D \cdot \sin$

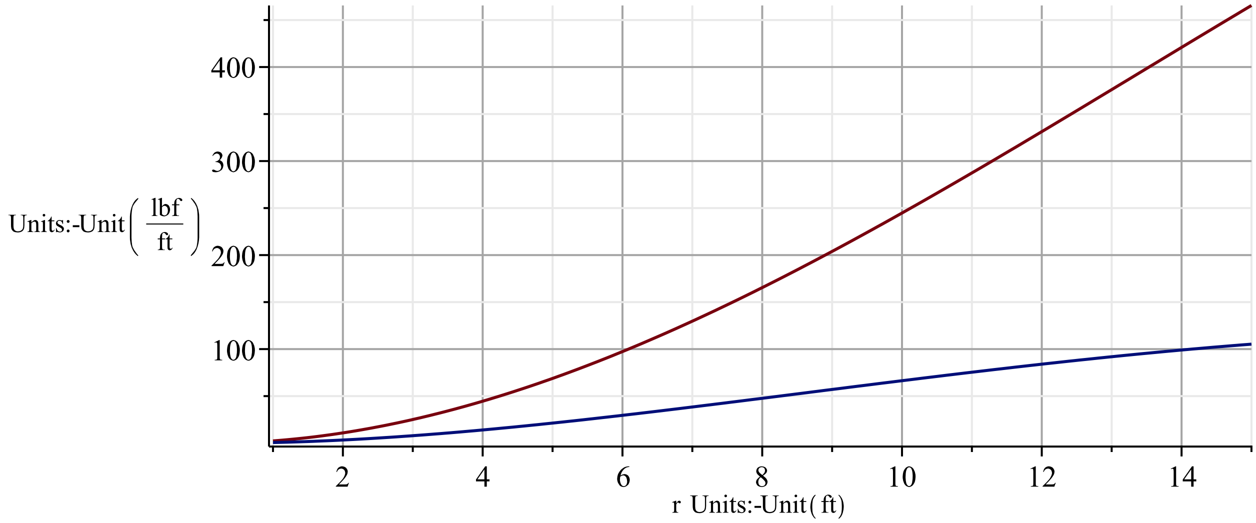
$$Rot \cdot LD = \begin{bmatrix} CC \cdot Ll - SS \cdot Dd \\ CC \cdot Dd + SS \cdot Ll \end{bmatrix}$$

$Q := L \cdot \sin + D \cdot \cos$

$th_s := (r, \theta, u) \rightarrow \mathrm{evalf}(lf_s(r, \theta, u) \cdot \cos(\alpha_i(r, \theta, u)) - dr_s(r, \theta, u) \cdot \sin(\alpha_i(r, \theta, u)))$

$trq_s := (r, \theta, u) \rightarrow \mathrm{evalf}(dr_s(r, \theta, u) \cdot \cos(\alpha_i(r, \theta, u)) + lf_s(r, \theta, u) \cdot \sin(\alpha_i(r, \theta, u)))$

$\mathrm{plot}\left(\left[th_s\left(r, 15deg, 5\frac{ft}{s}\right), trq_s\left(r, 15deg, 5\frac{ft}{s}\right)\right], r = R_{hng}..R_{tip}, \text{gridlines}, \text{useunits} = \left['ft', '\frac{lbf}{ft}'\right]\right) =$



$tm(4) := \mathrm{time}()$

$tm = [\ 4.781 \quad 5.359 \quad 6.656 \quad 11.187 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]$

These functions are then integrated along the radius to develop lift and torque:

$Thust := (\theta, u) \rightarrow N_{bld} \cdot \mathrm{int}(th_s(r, \theta, u), r = R_{hng}..R_{tip}, \text{numeric})$ $\quad Thust\left(15deg, 5\frac{ft}{s}\right) = 10547.025\ lbf$

$Torq := (\theta, u) \rightarrow N_{bld} \cdot \mathrm{int}(r \cdot trq_s(r, \theta, u), r = R_{hng}..R_{tip}, \text{numeric})$ $\quad Torq\left(15deg, 5\frac{ft}{s}\right) = 29561.234\ ft\ lbf$

$Thust\left(5deg, 10\frac{ft}{sec}\right) = 2844.7\ lbf$

$Thrst\left(5deg, 10\frac{ft}{sec}\right) = 2854.1\ lbf$

$tm(5) := \mathrm{time}()$

Down flow dynamic force $\quad UD := u \rightarrow \frac{\rho_{air} \cdot (2 \cdot u)^2}{2} \cdot \pi \cdot R_{tip}^{\ 2}$

$TH5 := u \rightarrow Thust(5deg, u)$

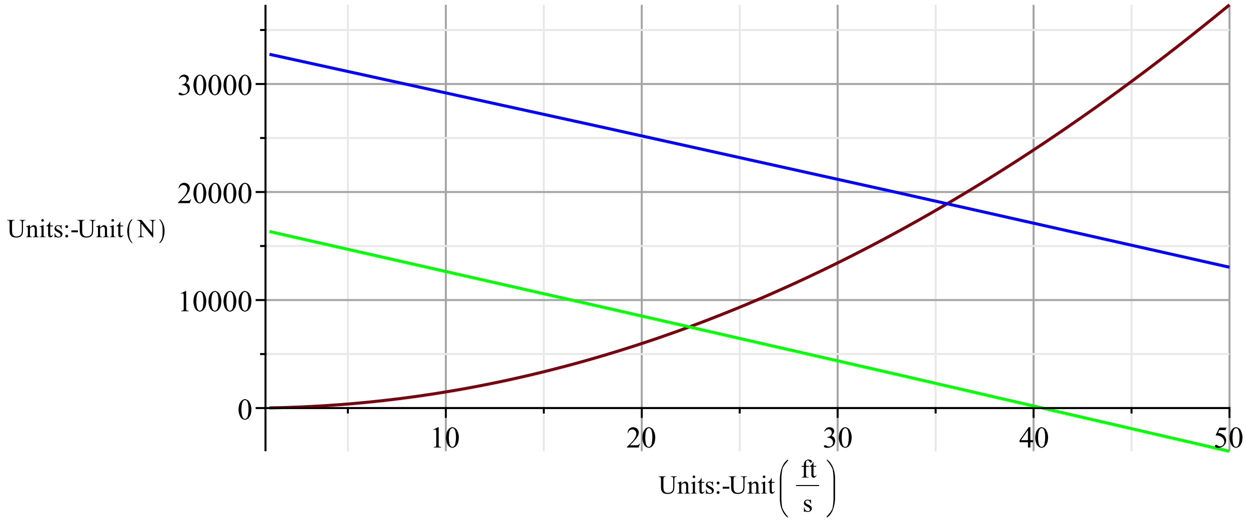
$TH10 := u \rightarrow Thust(10deg, u)$

$tm = [\ 4.781 \quad 5.359 \quad 6.656 \quad 11.187 \quad 14.421 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]$

$pltU := \mathrm{plot}\left(UD, 1\frac{ft}{s}..50\frac{ft}{s}, \text{gridlines}, \mathit{adaptive} = \text{false}\right)$

$pltT := \mathrm{plot}\left([TH5, TH10], 1\frac{ft}{s}..50\frac{ft}{s}, \text{color} = [\text{green}, \text{blue}], \text{gridlines}, \mathit{adaptive} = \text{false}, \text{numpoints} = 4\right)$

plots:-display(pltU, pltT) =



tm(6) := time( )

tm = [4.781 5.359 6.656 11.187 14.421 24.828 0 0 0 0]

Solving for the inflow velocity

The thrust of the rotor generates a dynamic pressure through the rotor, the dynamic pressure times the rotor area equals the thrust. The inflow velocity is twice the dynamic pressure velocity. **Note that the lines of thrust (Thust) as functions of downflow velocity at different blade angles appear to be parallel straight lines.** What is the slope of those lines?

$eqn2 := \theta \rightarrow Thust(\theta, u) = 2 \cdot \rho_{air} \cdot u^2 \cdot \pi \cdot R_{tip}^2$

$U := \theta \rightarrow fsolve\left(eqn2(\theta), u = 0\frac{ft}{s} .. 100\frac{ft}{s}\right)$

The two statements above should have solved for the inflow velocity that satisfies that condition, but Flow refuses to resolve them to an answer. Try to develop that solution, the point of intersectionin the graph above.

$Thust\left(5deg, 22\frac{ft}{s}\right) - 2 \cdot \rho_{air} \cdot \pi \cdot R_{tip}^2 \cdot \left(22\frac{ft}{s}\right)^2 = 106.312\ lbf$

$eps := 0.01$

$Fn := (\theta, u) \rightarrow Thust(\theta, u) - 2 \cdot \rho_{air} \cdot \pi \cdot R_{tip}^2 \cdot u^2$

$eps = 0.033\ \frac{ft}{s}$

The root equation: $Fn\left(5deg, 22.5\frac{ft}{s}\right) = -14.932\ lbf$

Derivative $DFn := (\theta, u, \delta) \rightarrow \frac{(Fn(\theta, u + \delta) - Fn(\theta, u - \delta))}{2 \cdot \delta}$

$$DFn\left(5deg, 22.5\frac{ft}{s}, 0.01\frac{ft}{s}\right) = -244.178\ \frac{lbf\ s}{ft}$$

$tm(7) := time()$

$$tm = \begin{bmatrix} 4.781 & 5.359 & 6.656 & 11.187 & 14.421 & 24.828 & 30.171 & 0 & 0 & 0 \end{bmatrix}$$

$$U := (\theta, u) \rightarrow u - \frac{Fn(\theta, u)}{DFn\left(\theta, u, 0.01\frac{ft}{s}\right)}$$

$$U\left(5deg, 25\frac{ft}{s}\right) = 22.524\ \frac{ft}{s}$$

$$U\left(5deg, 22.524\frac{ft}{s}\right) = 22.439\ \frac{ft}{s}$$

$$U\left(5deg, 22.439\frac{ft}{s}\right) = 22.439\ \frac{ft}{s}$$

$$U\left(15deg, 25\frac{ft}{s}\right) = 51.705\ \frac{ft}{s}$$

$$U\left(15deg, 51.705\frac{ft}{s}\right) = 46.121\ \frac{ft}{s}$$

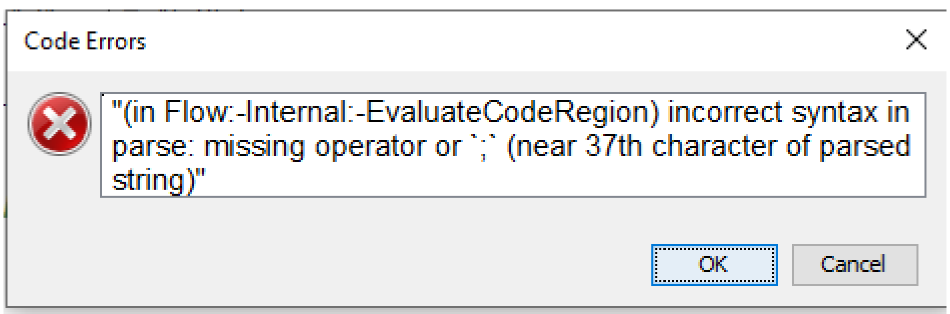
$$U\left(15deg, 46.121\frac{ft}{s}\right) = 45.853\ \frac{ft}{s}$$

$$U\left(15deg, 45.853\frac{ft}{s}\right) = 45.852\ \frac{ft}{s}$$

$$Thust\left(5deg, 22.439\frac{ft}{s}\right) = 1690.244\ lbf$$

$$Thust\left(15deg, 45.853\frac{ft}{s}\right) = 7057.799\ lbf$$

So how do I automate this? I tried to create a proceedure, and failed!



$$RnU := (\theta, u) \rightarrow U(\theta, U(\theta, U(\theta, U(\theta, U(\theta, u)))))$$

$$RnU\left(5deg, 30\frac{ft}{s}\right) = 22.438798\ \frac{ft}{s}$$

tm(8) := time( )

tm = [4.781   5.359   6.656   11.187   14.421   24.828   30.171   74.843   0   0]

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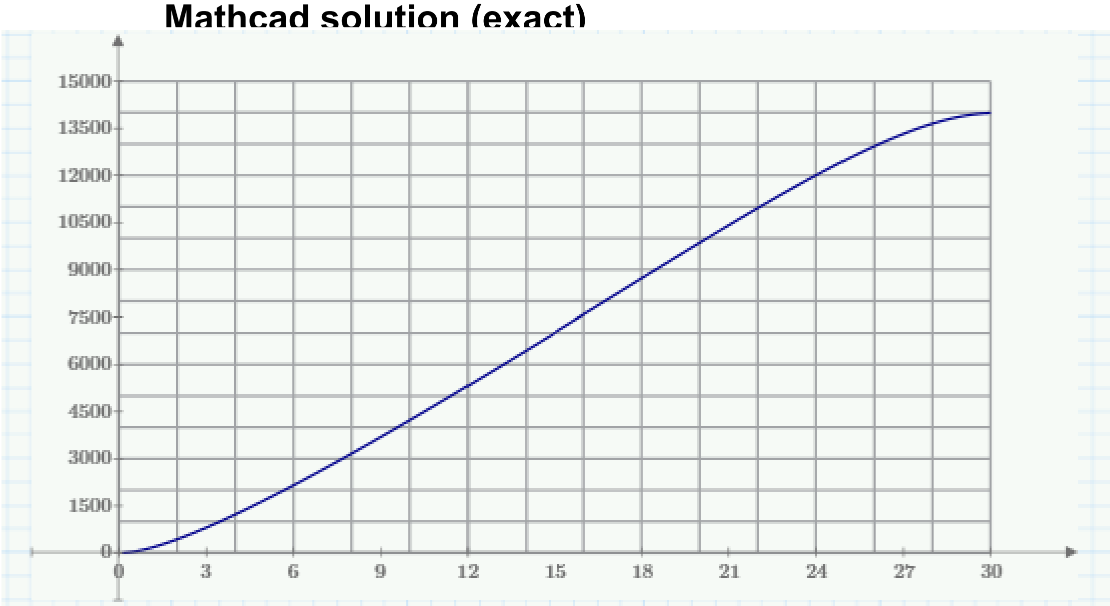
tm = [4.781   5.359   6.656   11.187   14.421   24.828   30.171   74.843   0   0]

tm(9) := time( )

tm = [4.781   5.359   6.656   11.187   14.421   24.828   30.171   74.843   74.921   0]

Mathcad solution (exact)



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