

Here our Graph G is always the circulant graph $G = \text{Circ}(n; \{d_1, d_2, \dots, d_k\})$. Then for the subgraph H of G we can have this type Labeling defined sometimes it may exist.

Recall that, for a sequence $\{d_1, d_2, \dots, d_k\}$ of positive integers with $1 \leq d_1 < d_2 < \dots < d_k \leq \lfloor \frac{n}{2} \rfloor$, the *circulant graph* $\text{Circ}(n; \{d_1, d_2, \dots, d_k\})$ has vertex set $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ and in which two vertices x and y being adjacent if and only if $x - y \equiv \pm d_i \pmod{n}$ for some i , $i \in \{1, 2, \dots, k\}$.

For an edge xy in $\text{Circ}(n; \{d_1, d_2, \dots, d_k\})$, the *length* of xy is $\min\{|x - y|, n - |x - y|\}$.

Given two edges $e_1 = u_1v_1$ and $e_2 = u_2v_2$ of same length ℓ in $\text{Circ}(n; \{d_1, d_2, \dots, d_k\})$, the *rotation-distance* $r(\ell)$ between e_1 and e_2 is $r(\ell) = \min\{r_1, r_2 : (u_1 + r_1)(v_1 + r_1) = e_2, (u_2 + r_2)(v_2 + r_2) = e_1\}$, where addition is reduced modulo n .

If $r(\ell) = \ell$, then the edges e_1 and e_2 are adjacent;

if $r(\ell) \neq \ell$, then e_1 and e_2 are nonadjacent.

An *orthogonal $\{1, 2, \dots, \lfloor \frac{n}{2} \rfloor\}$ -labelling* and generalized it to an orthogonal $\{d_1, d_2, \dots, d_k\}$ -labelling, where $\{d_1, d_2, \dots, d_k\}$ is a sequence of positive integers with $1 \leq d_1 < d_2 < \dots < d_k \leq \lfloor \frac{n}{2} \rfloor$.

I. Either n is odd or n is even and $d_k \neq \frac{n}{2}$:

Given a subgraph G of $\text{Circ}(n; \{d_1, d_2, \dots, d_k\})$ with $2k$ edges, a 1-1 mapping $\psi : V(G) \rightarrow \mathbb{Z}_n$ is an *orthogonal $\{d_1, d_2, \dots, d_k\}$ -labelling* of G if:

(i) for every $\ell \in \{d_1, d_2, \dots, d_k\}$, G contains exactly *two* edges of length ℓ , and

(ii) $\{r(\ell) : \ell \in \{d_1, d_2, \dots, d_k\}\} = \{d_1, d_2, \dots, d_k\}$.

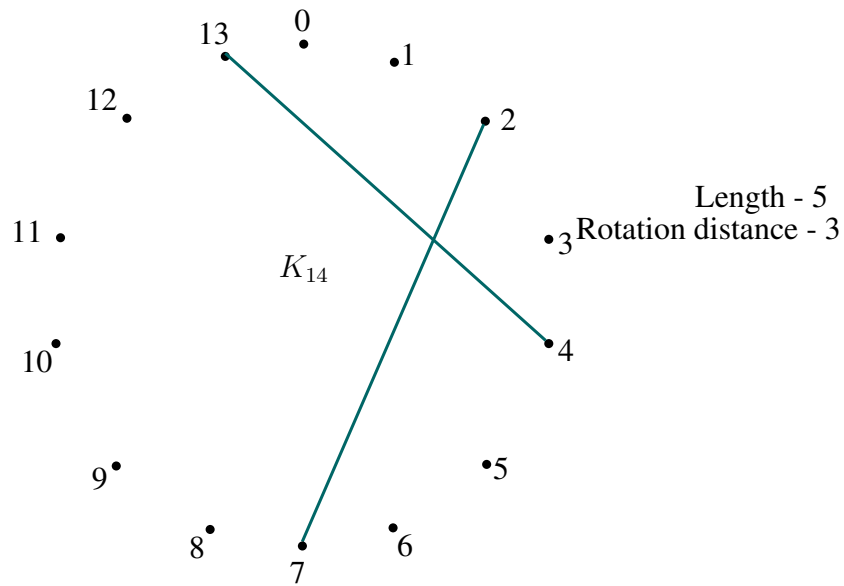
II. n is even and $d_k = \frac{n}{2}$:

Given a subgraph G of $\text{Circ}(n; \{d_1, d_2, \dots, d_{k-1}, \frac{n}{2}\})$ with $2k - 1$ edges, a 1-1

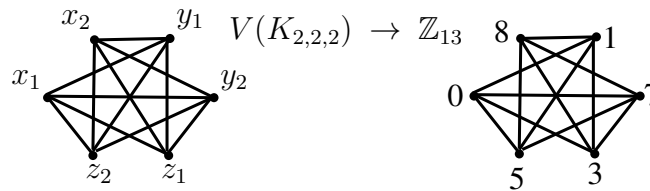
mapping $\psi : V(G) \rightarrow \mathbb{Z}_n$ is an *orthogonal* $\{d_1, d_2, \dots, d_{k-1}, \frac{n}{2}\}$ -labelling of G if:

- (i) for every $\ell \in \{d_1, d_2, \dots, d_{k-1}\}$, G contains exactly *two* edges of length ℓ , and G contains exactly *one* edge of length $\frac{n}{2}$, and
- (ii) $\{r(\ell) : \ell \in \{d_1, d_2, \dots, d_{k-1}\}\} = \{d_1, d_2, \dots, d_{k-1}\}$.

The addition is on modulo n .



An OL of $K_{2,2,2}$.



Edges	length	Rotation distance
$\{7, 8\}; \{0, 1\}$	1	6
$\{1, 3\}; \{5, 7\}$	2	4
$\{0, 3\}; \{5, 8\}$	3	5
$\{1, 5\}; \{3, 7\}$	4	2
$\{0, 5\}; \{3, 8\}$	5	3
$\{7, 0\}; \{8, 1\}$	6	1