Technically the curve is the set of points $C = \{\, (f(t), g(t)) \,|\, t \in [a, b] \,\}$. The functions f and g are called the x and y coordinate functions, respectively. The independent variable t is called the parameter and often represents time. For each $t \in [a, b]$ there is a corresponding point in the plane with coordinates $(f(t), g(t))$. Plotting these points as t varies over $[a, b]$ gives a collection of points in the plane which constitutes the curve. When t represents the time, the point $(f(t), g(t))$ represents the position of a particle moving in the plane and as t varies from a to b, the particle traces out the curve, which is also known as the trajectory of the particle. Note that in this sense, a parameterized curve has a direction associated to it, i.e., the direction in which the particle moves on its trajectory.

Also recall from calculus that a parameterized curve can alternatively be thought of as a vector-valued function $\mathbf{r} : [a, b] \to \mathbb{R}^2$, expressed by the formula

$$\mathbf{r}(t) = [f(t), g(t)],$$

for $t \in [a, b]$. Different calculus texts will use different notation for the component expressions for vectors. Here we use square brackets: $\mathbf{v} = [v_1, v_2]$, since this is how Maple represents vectors, namely, as lists, or 1-dimensional arrays. From a physical point of view, $\mathbf{r}(t)$ is the position vector of the particle at time t. While a vector can be plotted anywhere in the plane, the position vector is usually plotted with its initial point at the origin. Then its terminal point is $(f(t), g(t))$.

The first and second derivatives of the vector-valued function $\mathbf{r}$ are also vector-valued functions and in physics are called the velocity and acceleration function for the moving particle:

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{r}'(t) = [f'(t), g'(t)] \\ \mathbf{a}(t) &= \mathbf{r}''(t) = [f''(t), g''(t)]. \end{aligned}$$

The vector $\mathbf{v}(t)$, when plotted with its initial point at $(f(t), g(t))$ is tangent to the curve. The vector $\mathbf{a}(t)$, when plotted with its initial point at $(f(t), g(t))$ should point in the direction in which the curve is curving.

You will be assigned one of the following curves to study:

(a) $\mathbf{r}(t) = [t^4 - t, 2t^3 - t^5]$, for $t \in [-1.55, 1.55]$.

(b) $\mathbf{r}(t) = [t^5 - t^2 + 1, 2t^3 - t^2 - t + 1]$, for $t \in [-0.8, 1.2]$.

(c) $\mathbf{r}(t) = [t^6 - t^2 + 1, t^3 - t^2 - t + 1]$, for $t \in [-1.1, 1.25]$.

(d) $\mathbf{r}(t) = [t^4 - t^2 + 1, 2t^2 - t + 1]$, for $t \in [-0.8..1.2]$.

(e) $\mathbf{r}(t) = [t^4 - t^3 + 1, 2t^3 - t^2 - t + 1]$, for $t \in [-0.8, 1.2]$.

(f) $\mathbf{r}(t) = [t^4 - t^3 + 1, 2t^4 - t^2 - t + 1]$, for $t \in [-0.8, 1.2]$.

(g) $\mathbf{r}(t) = [t^5 - t^2 + 1, 2t^4 - t^2 - t + 1]$, for $t \in [-0.8, 1.2]$.

(h) $\mathbf{r}(t) = [t^5 - t^2 + 1, 2t^5 - t^2 - t + 1]$, for $t \in [-0.8, 1.2]$.