

- (i) $\mathbf{r}(t) = [t^5 - t + 1, 2t^6 - t^2 - t + 1]$, for $t \in [-1, 1.2]$.

For your particular curve do the following:

- (i) Study the way the velocity and acceleration change as the curve is swept out. Do this by choosing a point $Q = \mathbf{r}(t_0)$ on the curve (your choice, but choose an "interesting" one) and then plotting the tangent (or velocity) lines and acceleration lines at the points $Q_i = \mathbf{r}(t_0 + i\Delta t)$, for $i = 0, 1, 2, 3$. For this choose a suitably small Δt so that the points Q_i are not too far apart, yet far enough apart to be distinguished from each other. Store the plots of the velocity and acceleration lines in two arrays `vel` and `accel`. For the velocity lines, you will have to experiment in order to choose a suitable length, but plot each as a line segment starting at its respective Q_i and extending in the "forward" direction. Use an array of colors `c[i], i=0..3` to render each velocity line in a different color. Do a similar thing for the acceleration lines. Plot all of these and the curve itself in the same picture. Mark the directions on the lines and curve and annotate the figure (by hand after printing out).
- (ii) Use `fsolve` to find all points of intersection of your curve with itself. Print out and annotate a figure with this information.
- (iii) Use `fsolve` to find all points of intersection of your curve with the curve:

$$\mathbf{r}(t) = [t^3 - t, t^4 - t^2],$$

for $t \in [a, b]$. You will have to select a suitable interval $[a, b]$ by experimentation. Plot both curves in the same picture and mark your answer on the printout of the picture.

2.8 Maple/Calculus Notes

We review here a number of topics from both calculus and Maple for dealing with plane curves. Recall that a plane curve can be represented in two ways: by an equation $H(x, y) = 0$ (the implicit description) and parametrically by a pair of equations involving a single parameter (the parametric description).

2.8.1 Planes Curves Given Implicitly

- (1) To define a function $H(x, y) = xy^2 - y^3 + 1$ of two variables in Maple use the arrow notation much as for a function of one variable:

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H:=(x,y)->x*y^2-y^3+1;
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