

```

A:= [0,1,2]; B:= [0,1,2];
T:= array([ [1,2], [1.5,1] ]);
g:=(x,y) → f(A,B,T,x,y);

```

defines a function  $g$  which coincides with the step function in the last example above.

**Example 4.7 (Conic Sections)** As another example of branching, we will look at the classification of conic sections given by the equation

$$Ax^2 + By^2 + Cx + Dy + E = 0. \quad (4.1)$$

Since there is no  $xy$  term, the conic sections will have horizontal and/or vertical axes of symmetry. We will organize our procedure around the coefficients of the square terms  $A$  and  $B$ . We have the following cases:

1. ( $A \neq 0$  and  $B \neq 0$ ) Completing the square on both of the squared terms in equation (4.1) yields

$$A\left(x + \frac{C}{2A}\right)^2 + B\left(y + \frac{D}{2B}\right)^2 = M,$$

where  $M = \frac{BC^2 + AD^2 - 4EAB}{4AB}$ .  $\rightarrow ABCDE$

(i) If  $M = 0$  and if

(a)  $A$  and  $B$  have the same sign, then the graph is just a point  $(-C/2A, -D/2B)$ .

(b)  $A$  and  $B$  have opposite signs, then the graph is two lines  $x + \frac{C}{2A} = \pm \sqrt{\left|\frac{B}{A}\right|} \left(y + \frac{D}{2B}\right)$ .

(ii) If  $M \neq 0$ , then the equation can be rewritten as

$$\frac{(x + C/2A)^2}{M/A} + \frac{(y + D/2B)^2}{M/B} = 1.$$

(a) If  $M/A$  and  $M/B$  are both positive, the graph is an ellipse.

(b) If  $M/A$  and  $M/B$  have opposite signs, the graph is a hyperbola.

(c) If  $M/A$  and  $M/B$  are both negative, there is no graph.

2. ( $A = 0$  and  $B \neq 0$ ) Equation (4.1) in this case is equivalent to

$$B\left(y + \frac{D}{2B}\right)^2 = -Cx - E + \frac{D^2}{4B}.$$